Abstract. This paper investigates the effects of network structures on asset price dynamics. We introduce network communication structures into a simple present value discounted asset pricing model with heterogeneous expectations. Every period the agents choose a predictor of the future price based on the past performance of their own and alternative predictors if they are observed and determine their demand for a risky asset. The information about the performance of alternative predictors...
is available only through locally connected agents. We model communication structure using four types of commonly considered networks: a fully connected network, a regular lattice, a small world, and a random network. The results show that the network structure influences asset price dynamics in terms of the regions of stability, amplitudes of fluctuations and their statistical properties.

**Keywords:** asset pricing, local interactions, networks, small world, heterogeneous beliefs, price dynamics.

**JEL classification:** C45, C62, C63, D84, G12.
1 Introduction

To a large extent innovative ideas and practices diffuse across communities through interpersonal communication. Popular ideas in financial markets also often spread through conversations (Shiller, 1995). In a survey of institutional investors in the USA, Shiller and Pound (1989) found that money managers who invested in stocks with extremely high growth of the price/earnings ratio were often discussing their trades with colleagues. Arnswald (2001) found that among fund managers in Germany information exchange with other financial and industry experts was the second most important factor influencing their investment decisions, complemented by conversations with their colleagues and reports from media. Similarly, a study of fund managers by Hong et al. (2005) provided strong support for the importance of informal communication. Cohen et al. (2008) provide empirical evidence that connections between mutual fund managers and corporate board members via shared education networks have a significant effect on mutual fund portfolio performance.

Household investment decisions are also affected by interpersonal communication. Madrian and Shea (2000) and Duflo and Saez (2002) showed that employees are more likely to join an investment retirement scheme if their colleagues have done so. Hong et al. (2004) suggested, by reviewing data from the University of Michigan health and retirement study, that interaction with neighbors and church attendance increased the likelihood of a household investing in stocks.

Given this evidence, in this paper, we study the impact that local interactions between investors have on the asset price dynamics in the theoretical model of asset pricing. We bring together ideas from various streams of literature: the rapidly developing literature on networks, the literature on heterogeneous agent models and agent-based models. We explore a range of local interaction patterns by introducing different types of network topologies into the stylized heterogeneous agent model of

We find that communication patterns influence asset price dynamics in terms of the regions of stability, amplitudes of fluctuations and statistical properties. However, their impact heavily depends on the considered agents’ ecologies.

In the next section we survey models with boundedly rational and heterogeneous agents. Section 3 examines different network structures and their properties. Section 4 contains the description of the model, its application to the model of Brock and Hommes (1998) and derives some analytical results for the random graph model. Section 5 uses simulations to analyze the model under various network structures, presents and discusses the results of the simulations. Section 6 concludes and discusses further extensions.

2 Bounded rationality and heterogeneity in asset pricing

The rational expectations theory in finance (Friedman, 1953) asserts that rational investors would drive irrational traders out of financial markets. Numerous empirical studies, however, have shown that successful traders follow a variety of investment strategies (e.g. Frankel and Froot, 1987; Ito, 1990). DeLong et al. (1990) was among the first studies to analytically demonstrate that irrational noise traders may survive in a market with fully rational traders. This survival is possible because these noise traders bear a higher risk which leads to higher returns in the long run. Other researchers used heterogeneity of expectations to explain asset prices dynamics. Day and Huang (1990), Chiarella (1992), Kirman (1993) and Lux (1995) showed that transactions between different agents that follow simple behavioral rules and interact with each other lead to endogenous price fluctuations. Alfarano and Milakovic (2009) enriched the Kirman-Lux model with explicit network structures. Anufriev and Bottazzi (2012) analyzed the implication of heterogeneity in investment
Brock and Hommes (1998) introduced a structural asset pricing model with heterogeneous agents (denoted the BH model henceforth) with evolutionary switching between several trading strategies. The strategies differed only in the expectation about the future price of the risky asset. The performance measure of each strategy was freely available to all agents. The BH model showed that the rational expectations strategies do not necessarily drive out boundedly rational strategies. In fact, both types can co-exist in a market. Brock and Hommes (1998) conducted stability analysis of the steady states and derived the conditions for occurrence of certain bifurcations. The BH model was able to produce excess volatility and positive volatility/volume correlations, the stylized facts which were not reproduced by the rational expectations models.

dynamics in macroeconomic setting. Anufriev and Hommes (2012a,b) introduced a BH-style heuristics switching model and estimated it using data from a learning-to-forecast experiment. For a detailed survey of the heterogeneous agent literature see Hommes (2006).

Another stream of asset pricing literature focuses on large-scale models of evolving, interacting artificial agents. Examples of this approach include the Santa Fe artificial stock market (Arthur et al., 1997; LeBaron et al., 1999; Ehrentreich, 2006) and the models of Chen and Yeh (2001) and Chen et al. (2001). A major advantage of these models over the smaller scale heterogeneous agent models previously discussed is that they allow for higher flexibility, richer behavioral assumptions, and more realistic market architectures. This, however, comes at the price of increased complexity. Analytical solutions are not typically attainable for these models and, therefore, computer simulations are often used to study their properties. The literature on agent-based finance has also been influenced by the contributions of interdisciplinary statistical physicists. In particular, Iori (2002) and Cont and Bouchaud (2000) explicitly considered network structures in their models of financial markets. For further details and references on agent-based finance we refer the interested reader to the review by LeBaron (2006).

In this paper we combine the rapidly developing work on networks with heterogeneous agent models by introducing local interactions into the stylized BH model. Our aim is to study the effects of different types of local interactions on the asset price dynamics. In our setting the information about the performance measure of each particular strategy is only available to the agents locally through their own experience and the experience of other agents directly connected to them. We explore a range of local interaction patterns by introducing different types of networks. We derive transition equations reflecting these local interactions and offer analytical approximations for some network types. When analytical approximations are not
available due to high complexity of the system, we rely on computer agent-based simulations to investigate the model.

3 Social networks

Social networks are important elements of our lives. Decision making, trade activity, job searching, disease transmission are all heavily influenced by the various structures of social and economic networks. Network modeling is a rapidly growing part of economic literature (see Jackson, 2008 for a detailed treatment). Watts (1999) indicates that a typical social network has the following properties: 1) there are many participants in the network; 2) each participant is connected to a small fraction of the entire network, i.e., the network is sparse; 3) even the most connected node is still connected only to a small fraction of the entire network, i.e., the network is decentralized; 4) neighborhoods overlap, i.e., the network is clustered; and yet 5) diameter of the network, i.e., the shortest path between the furthest pair of nodes, is small.

To capture these properties Watts and Strogatz (1998) introduced a network model called a small world. It is an intermediate network between a regular lattice network, where the agents (nodes) are connected in a geometrically regular way, and a random graph, where the connections are random. Social scientists have recognized that the small world network is a good model approximating social interactions in real life. Networks with the small world properties include social networks of the US corporate elite (Davis et al., 2003), partnerships of investment banks in Canada (Baum et al., 2003), and many more. Small world networks emerge when participating agents form networks through a mix of random and strategic interactions (Baum et al., 2003 and Morone and Taylor, 2004).

Figure 1 shows four examples of network topologies. In the fully connected
network, all nodes are linked to all other nodes. In the regular lattice, each node is connected to two nodes on each side, that is, each node has $K = 4$ connections, or edges. In order to form a small world network an edge is reconnected to a different randomly chosen node on the lattice (avoiding self- and double-connection) with a given rewiring probability, $0 < \pi < 1$. Such rewiring of the nodes continues until all the edges are processed. In the limit when $\pi = 1$ the network becomes the random graph of Erdős and Rényi (1959), with $N$ nodes and probability of connection between any two nodes approximately equal to $K/N$.

The structural properties of a network can be quantified in terms of three additional characteristics (Newman, 2003): degree distribution, a clustering coefficient, $C$, and a characteristic path length, $L$. The degree distribution is the distribution of the number of edges, or the size of the immediate neighborhood, for each node. The average degree for the Watts and Strogatz (1998) model is equal to the number of edges $K$ in the initial regular lattice for all $\pi$. The clustering coefficient of a node is calculated by dividing the number of edges between the direct neighbors of this node by the maximum possible number of edges between them. It indicates how well the neighborhood of the node is connected or, in other words, it expresses the ‘cliquishness’ of the neighborhood. By averaging over the clustering coefficients of all the nodes in a network we obtain the clustering coefficient of the network $C$. In
the Watts and Strogatz (1998) model $C$ is increasing in $K$ and decreasing in $\pi$. The characteristic path length $L$ measures the average separation between two nodes and is defined as the average number of edges in the shortest path between two nodes. In the Watts and Strogatz (1998) model $L$ is increasing in $N$ and decreasing in $K$ and $\pi$ (see Newman (2003) for detailed discussion on relations between these values).

Latora and Marchiori (2001) relate the clustering coefficient and the inverse of the characteristic path length to the local and global efficiency of the network, respectively. Local efficiency measures fault tolerance, that is, how efficient is the communication between the immediate neighbors of node $i$, when $i$ is removed. Global efficiency is related to the signal transmission through the whole network.

For each value of the rewiring probability, $\pi$, we obtain a network with new structural properties. These properties also depend on $N$ and $K$. A small world network can be formally defined in terms of the clustering coefficient and characteristic path length. Specifically, it is a decentralized, sparsely connected network with a high clustering coefficient $C$ and a small characteristic path length $L$. The values of the normalized clustering coefficients and the characteristic path lengths for different rewiring probabilities $\pi$ and two network sizes with $N = 100$ and $N = 1000$ nodes are depicted in Figure 2. Normalization is implemented over the correspond-
ing characteristics of the regular lattice (for which $\pi = 0$). The small world network properties emerge in a setting when $\pi = 0.1$ for $N = 100$ and $\pi = 0.01$ for $N = 1000$. This is consistent with Albert and Barabási (2002) who suggest that the rewiring probability leading to a small world network is inversely proportional to the number of nodes, $N$.

The Watts and Strogatz (1998) model is a popular choice in the social networks literature, but as with any model it has some limitations. One of its main drawbacks is that the model is unable to produce the degree distribution observed in typical real social networks. Scale free networks suggested by de Solla Price (1965) and recently advanced by Barabási and Albert (1999) address this problem, but are often unable to generate realistic clustering as observed in social networks. Hence, in this paper, we limit ourselves to the Watts and Strogatz (1998) networks.

4 Heterogeneous belief model with local interactions

4.1 Brock-Hommes model

In this section we first describe the BH model, and then extend it by allowing for local interactions. There are two assets that are traded in discrete time: a risk-free asset paying a constant gross return, $R = 1 + r$, and a risky asset paying a stochastic dividend, $y_t$, at the beginning of each trading period $t$. The dividend is assumed to be independently and identically normally distributed (i.i.d.) with mean $\bar{y}$ and variance $\text{Var}[y]$. The price, $p_t$, per-share (ex-dividend) of the risky asset in period $t$ is obtained from the Walrasian market clearing condition. The wealth dynamics is specified by

$$W_{t+1} = R(W_t - p_t z_t) + (p_{t+1} + y_{t+1}) z_t = RW_t + (p_{t+1} + y_{t+1} - Rp_t) z_t,$$  \hspace{1cm} (1)

\footnote{While the BH model is presented in terms of deviations from the fundamental price, we present the model in terms of the price itself for better exposition.}
where $W_t$ and $W_{t+1}$ are the wealth levels in period $t$ and $t+1$ correspondingly, and $z_t$ is the number of shares of the risky asset purchased at date $t$.

The agents are myopic maximizers of the mean-variance expected wealth:

$$\max_{z_t} \left\{ E_{t-1}[W_{t+1}] - \frac{a}{2} V_{t-1}[W_{t+1}] \right\},$$

where $a$ is the absolute risk aversion coefficient, and $E_t$ and $V_t$ denote conditional expectation and conditional variance that are based on the publicly available information set $I_t = \{p_t, p_{t-1}, p_{t-2}, \ldots; y_t, y_{t-1}, y_{t-2}, \ldots\}$. There are $H$ belief types which differ in their expectations about the future price. The demand for the risky asset of type $h$ is then given by:

$$z^h_t(p_t) = \frac{E^h_{t-1}[p_{t+1} + y_{t+1}] - Rp_t}{a V^h_{t-1}[p_{t+1} + y_{t+1}]} = \frac{E^h_{t-1}[p_{t+1} + y_{t+1}] - Rp_t}{a \sigma^2}.$$

Operators $E^h_{t-1}[\cdot]$ and $V^h_{t-1}[\cdot]$ are the expectations or predictors of type $h$ of the mean and the variance, respectively. The predictors for period $(t+1)$ depend on $(t-1)$ information because the price at period $t$ is not realized at the moment the predictors are produced and actually depends on the value of the predictors (see Figure 3 for timing in the model). It is assumed that all the types expect the same variance, $V^h_t = \sigma^2$ and have the same value for the risk-aversion coefficient, $a$.

Set the supply of outside shares of the risky asset to zero.

$$\sum_{h=1}^H n^h_t \frac{E^h_{t-1}[p_{t+1} + y_{t+1}] - Rp_t}{a \sigma^2} = 0.$$  

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2This is a standard assumption of the baseline BH model. Hommes et al. (2005) consider a positive supply of the risky asset.
Under the assumption of homogeneous beliefs \((H = 1)\), the fundamental price, \(p^*\) is the unique constant solution to the market-clearing equation (4). It is equal to the discounted infinite sum of the expected future dividends, i.e., \(p^* = \bar{y}/r\).

All beliefs are of the form

\[
E_h^{t-1}[p_{t+1} + y_{t+1}] = b^h + p^* + \bar{y} + g^h(p_{t-1} - p^*),
\]

where \(b^h\) is a constant bias and \(g^h\) is an extrapolation parameter.

The main focus of this paper will be on the ecology with two types \((H=2)\), **fundamentalists** and **chartists** (Section 4.1.2 of Brock and Hommes, 1998). Both of these types have zero bias, \(b = 0\). Fundamentalists believe that price will be at the fundamental level, \(p^*\), and set the extrapolation parameter, \(g\), to 0, while chartists expect persistent deviations from the fundamental value and use a positive extrapolation parameter, \(g > 0\). In Section 5.3 this ecology is extended to four types by introducing positively and negatively biased types (Section 4.3 of Brock and Hommes, 1998).

Define the performance measure, \(U_h^t\), as a net profit of type \(h\), that is

\[
U_h^t = (p_t + y_t - Rp_{t-1})z_{t-1} - c^h,
\]

where \(p_t + y_t - Rp_{t-1}\) is the excess return earned per unit of the risky asset, \(z_{t-1}\), held in the agents’ portfolio at the end of period \(t-1\), and \(c^h\) is the cost of following the strategy of type \(h\). In the ecology with two types this cost is set to zero for the extrapolating rule of chartists and is strictly positive for fundamentalists.\(^3\) Costs are set to 0 for all types in the ecology with four types.

The belief types are updated over time depending on the relative utility from \(^3\)This is a standard assumption of the two-type BH model. Brock and Hommes (1998) suggest attributing these costs to ’training’ costs required to understand the theory behind the fundamentalists strategy.
following a strategy of a specific type compared to other types. The utility is based on the observed performance measure, $U_t^h$, and an unobserved idiosyncratic random component, $\varepsilon_t^h$, that is

$$\tilde{U}_t^h = U_t^h + \frac{1}{\beta} \varepsilon_t^h,$$

(7)

where $\beta$ is the intensity of choice parameter, which controls the level of the random component. The sources of randomness in the satisfaction are unobserved variations in preferences of agents and in the attributes of alternatives, and agents’ errors of perception and behavioral biases (Hirshleifer, 2001). In case when the (noisy) performances of all types are observed by all agents, the probability that an agent selects type $h$ at period $t$ is given by $P_t^h = P(\tilde{U}_t^h > \tilde{U}_t^k, \text{for all } k \neq h)$.

For a sufficiently large number of the agents, the fraction of agents of type $h$, $n_t^h$, converges to probability $P_t^h$. Moreover, if we assume that the idiosyncratic random component, $\varepsilon_t^h$, in (7) follows the standard Gumbel (extreme value) distribution, $n_t^h$ can be described by a discrete choice logit model (Manski and McFadden, 1990):

$$n_t^h = \frac{\exp(\beta U_t^h)}{\sum_{\xi=1}^{H} \exp(\beta U_t^\xi)}.$$  

(8)

The dynamics of the model is described through the co-evolution of the fractions of the types, and the market equilibrium price.

4.2 Local interactions

In our setup the agents are located on the nodes of a network and can observe the performance measure of the predictor types employed only by those agents who reside on the nodes directly connected with them. Hence, they cannot observe the performance of the types adopted by agents located two or more edges away.

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4If we assume Normal distribution for the idiosyncratic random component, Probit model will arise instead.
Therefore, contrary to the BH model, we do not assume that the performance of every type is available to all the agents. Instead, we allow only for local information exchange in the market.

In particular, if an agent is directly connected only to the agents of the same type, they are not able to switch as there is no information about the performance of the alternative type(s). If an agent has at least one neighbor of a different type, they are able to compare the utility from their own type with the utility from the alternative observed type(s) and make a choice. Note that under local information exchange, the fractions of the belief types, \( n_t^h \), do not follow the discrete choice fractions specified in Eq. 8 because some agents are not able to switch.

The described model does not pretend to fully represent real financial markets, but, perhaps, it is instructive to set a scene in which this model and, in particular, local interactions are important. Suppose that our world is populated by many individuals who invest their money (to save money for their retirement) in mutual (pension) funds. There is a limited number of funds to choose from, i.e., two in the two-type model and four in the four-type model. These funds consistently adopt a certain predictor, i.e., in the case of the two-type model, “fundamentalists” and “chartists”. Moreover, in the two-type model, the fundamentalist fund charges higher fees. The individuals are not professional investors and do not know or understand the predictor used by their fund. Every period, say, once in a quarter, the individuals receive reports on the performance of their funds. They are allowed to flexibly choose and change their funds. The individuals interact with their friends and if their friends are with a different fund, they are able to compare the relative

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5If there is more than one neighbor of the different type, only one of them is consulted to compare utilities. A more general setting is discussed in model extensions.

6To generalize further we able to assume that there are many funds which can be broadly classified into several types. Any differences of strategies within one type will have to be attributed to the additional unobserved random component. Note that the random component will affect both the pricing equation and the performance measure. Only switching between the different types of funds will be observed by the model.
performance of two funds and possibly switch.

The model progresses in the following way (see Figure 3). After the expectations of different types (of funds) are formed and the demands are ascertained, trade occurs and market clears. Next, the performances are released. Then, agents (investors) compare their utility with their neighbors’ and switch to another type (of fund) or remain with their current type depending on relative performance. Finally, the expectations are formed again and the cycle repeats.

For simplicity, we consider next the model with two types and denote these types by $A$ and $\bar{A}$. Denote the probability that agent $i$ is of type $A$ (invests in the fund which adopts the predictor of type $A$) at period $t$ by $P_{i,t}$. Then, the probability of being the other type, $\bar{A}$, is simply $(1 - P_{i,t})$.

The evolution of $P_{i,t}$ can be described by

$$P_{i,t} = P_{i,t-1} \prod_{j \in G_i} P_{j,t-1} + \left[ P_{i,t-1} \left( 1 - \prod_{j \in G_i} P_{j,t-1} \right) + (1 - P_{i,t}) \left( 1 - \prod_{j \in G_i} (1 - P_{j,t-1}) \right) \right] \Delta_t,$$

(9)

where $G_i$ denotes a neighborhood of agent $i$, the set of agents directly connected to $i$, excluding $i$, and $\Delta_t = (1 + \exp[\beta(U_{iA}^t - U_{i\bar{A}}^t)])^{-1}$ is the discrete choice logistic probability of choosing type $A$ over type $\bar{A}$ when both performances are observed. The first component of Eq. 9 determines the probability that agent $i$ is of type $A$ in
period \( (t-1) \) and is unable to switch because she is surrounded by the neighbors of the same type, while the second component consists of two parts (multiplied by \( \Delta_t \)). The first part determines the probability that agent \( i \) is of type \( A \), is neighbored by at least one agent of type \( \bar{A} \) and, hence, is able to choose between \( A \) and \( \bar{A} \). The second part determines the probability that agent \( i \) is of type \( \bar{A} \), is neighbored by at least one agent of type \( A \) and, hence, also is able to choose between \( A \) and \( \bar{A} \). The extension of the model to more than two types is conceptually straightforward, but tedious. We would have to keep track of all possible permutations where an agent has (partial) information about performance of types which belong to various subsets of all possible types and consider multiple \( \Delta \)-s depending on these subsets.

Note that Eq. 9 resembles the BH model with asynchronous updating (Hommes et al., 2005), where a fraction of one type is determined by 
\[
\begin{align*}
n_t &= \alpha_1 n_{t-1} + \alpha_2 \Delta_t, \\
0 &\leq \alpha_{1,2} \leq 1, \alpha_1 + \alpha_2 = 1.
\end{align*}
\]
Weights \( \alpha_1 \) and \( \alpha_2 \) determine fractions of agents who (1) keep their previous type and (2) are able to choose between the two types, respectively. The difference is that in our case the weights, \( \alpha_{1,2} \), are state- and agent-dependent, time varying, determined by the network topology and do not add up to 1. In the context of the BH model with two types, values of \( \beta \) for which bifurcations are observed do not typically change when an asynchronous updating is introduced. As we show below in our model local information exchange leads to significant quantitative and qualitative implications including changing values of \( \beta \) for which bifurcations are observed.

For a general network topology the probability of being a certain type are agent \( i \) specific because the neighbors of \( i \) may be directly connected between themselves (clustered neighborhood \( G_i \)), which makes the probabilities of them being of a certain type correlated. Therefore we are not able to represent the fraction of a specific type by this probability. There are two cases, however, where we are able to do so, and reduce the dimensionality of the resulting system. For the fully connected graph,
where all agents are in one large neighborhood and the number of agents is large \((N \to \infty)\), degree \(k \to \infty\), all products, \(\prod\)-s, in Eq. 9 tend to 0 and we recover the original BH model with \(n_t = \Delta_t\). In the random network, where connections between agents are random and the neighborhoods are not clustered we are able to drop individual agent indices \(i\) and \(j\) in Eq. 9 and for sufficiently large number of agents describe the evolution of fractions by

\[
 n_t = n_{t-1}n_t^k + [n_{t-1}(1 - n_{t-1}^k) + (1 - n_{t-1})(1 - (1 - n_{t-1})^k)]\Delta_t = \\
 n_{t-1}^{k+1} + [1 - n_{t-1}^{k+1} - (1 - n_{t-1})^{k+1}]\Delta_t, \tag{10}
\]

where \(k\) denotes the (average) degree of the network. As we have discussed in the previous section, for the random graph based on the Watts and Strogatz (1998) model, \(k = K\), where \(K\) is the number of edges in the initial regular lattice.

Note that Eq. 10 resembles the BH model with asynchronous updating (Hommes et al., 2005), where a fraction of one type is determined by \(n_t = \alpha_1 n_{t-1} + \alpha_2 \Delta_t\), \(0 \leq \alpha_{1,2} \leq 1, \alpha_1 + \alpha_2 = 1\). Weights \(\alpha_1\) and \(\alpha_2\) determine fractions of agents who (1) keep their previous type and (2) are able to choose between the two types, respectively. The difference is that in our case the weights, \(\alpha_{1,2}\), are state-dependent, time varying, determined by the network topology and do not add up to 1. In the context of the BH model with two types, values of \(\beta\) for which bifurcations are observed do not typically change when an asynchronous updating is introduced. As we show below in our model local information exchange leads to significant quantitative and qualitative implications including changing critical values of \(\beta\) for which bifurcations are observed.

Eqs. 3–6 and 10 jointly determine a system of difference equations governing the dynamics of the system. The analysis of the system is easier when the price is written in terms of deviations from the fundamental price, \(x_t = p_t - p^*\). Specifically,
$n_t = n_{t-1}^{k+1} + (1 - n_{t-1}^{k+1} - (1 - n_{t-1})^{k+1})\Delta t$

$E^\circ = (0, n^\circ)$, there are there two additional fun-
Table 1: Possible steady states and their stability for the system specified by Eqs. 11–12.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$\beta &lt; \ln(k)/c$</th>
<th>$\beta &gt; \ln(k)/c$</th>
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</thead>
<tbody>
<tr>
<td>$0 &lt; g &lt; R$</td>
<td>$E_0, E_2, E_1$ exist</td>
<td>$E_0$ is unstable</td>
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<td></td>
<td>$E_0, E_1$ are unstable</td>
<td>$E_0$ is unstable</td>
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<tr>
<td>$g &gt; 2R$</td>
<td>$E_0, E_0, E_1, E_+,$ $E_-$ exist</td>
<td>$E_0, E_1, E_+,$ $E_-$ exist</td>
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<tr>
<td></td>
<td>$E_0, E_0, E_1$ are unstable</td>
<td>$E_0, E_1$ are unstable</td>
</tr>
<tr>
<td>$R &lt; g &lt; 2R$ and $n^* &gt; n^*$</td>
<td>$E_0, E_0, E_1$ exist</td>
<td>$E_0$ is unstable</td>
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<td>$E_0, E_1$ are unstable</td>
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<tr>
<td>$R &lt; g &lt; 2R$ and $n^* &lt; n^*$</td>
<td>$E_0, E_0, E_1, E_+,$ $E_-$ exist</td>
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<td>$E_0, E_0, E_1$ are unstable</td>
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Fundamental steady states, $E_0 = (0, 0)$ and $E_1 = (0, 1)$. Note that $n^*$ is increasing in $\beta c$, and reaches the value of 1 when $\beta c = \ln k$ (see Figure 4a). The stability conditions of the steady states are determined by analyzing the eigenvalues at the corresponding steady states. For the fundamental steady state we get one zero eigenvalue, another eigenvalue equal to $(g/R)n^*$ (these two are the same as in the original BH model) and an additional eigenvalue equal to $\frac{(k+1)\exp(\beta c)(1-n^*)^k+(n^*)^k}{1+\exp(\beta c)}$. Steady state $E_0$ is always unstable, steady state $E_1$ is unstable when $\beta c < \ln k$. Let $n^* = R/g$ and $x^*$ be the positive solution (if it exists) of $n^* = n^* k^+1 + \frac{1-(n^*)^k+1-(1-n^*)^k+1}{1+\exp(\beta c)}$. Define as in the BH model, two non-fundamental steady states $E_+ = (x^*, n^*)$, $E_- = (-x^*, n^*)$. By relying on the results of Lemma 2 in Brock and Hommes (1998) and the graphical analysis of the above steady state equations and stability conditions, we conjecture several possible scenarios which are summarized in Table 1.

The case when $R < g < 2R$ is of the most interest and we discuss it in more details. At $\beta = 0$ or $c = 0$ the fraction of chartists at the stable fundamental steady state, $n^*$ is 1/2. As parameter $\beta$ increases $n^*$ increases as well. At some point $\beta = \beta^*$ which solves $n^* = n^* k^+1 + \frac{1-(n^*)^k+1-(1-n^*)^k+1}{1+\exp(\beta c)}$ we conjecture the occurrence of the primary pitchfork bifurcation. For $\beta > \beta^*$, the fundamental steady state, $E_0$, becomes unstable and the non-fundamental steady states, $E_+, E_-$, arise.
As $\beta$ increases further and reaches some point, $\beta^{**}$, we conjecture the occurrence of the secondary Neimark-Sacker bifurcation in a way similar to the original BH model (Lemma 3, Brock and Hommes, 1998). For $\beta > \beta^*$, the non-fundamental steady states, $E_+, E_-$, lose their stability. Our conjectures are supported by the numerical analysis carried out in the E&F Chaos (Diks et al., 2008). Both $\beta^*$ and $\beta^{**}$ are increasing in the (average) degree of the network, $k$, as shown in Figure 4b for a typical two-type BH specification with parameters $g = 1.2, R = 1.1, c = 1, D = 1$. As the value of $k$ is increasing the values of $\beta^*$ and $\beta^{**}$ are approaching the bifurcation values in the original BH model.

We have previously mentioned that the convenient low dimensional representation analyzed above cannot be easily derived for networks with clustered neighborhoods such as the regular lattice or the small world networks. Intuitively, having directly connected neighbors with correlated types decreases the informational content of the neighborhood. In the context of the random graph network this may be viewed as a reduction in the “effective” neighborhood size or degree of the network $k$. Hence, by fixing the “nominal” value of $k$, we may expect earlier (for smaller $\beta$) bifurcations for more clustered networks when compared to the random networks.

We proceed with agent-based simulations to investigate whether this prediction holds and, more generally, to compare the effects of various network topologies. In the next section we also provide a more intuitive description of the observed bifurcations and dynamic behavior of the price.

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7The E&F chaos code for this model and the generated plots are available on request.
5 Simulations and results

We conduct numerical simulations\(^8\) for four different network structures of local interactions, i.e. for a fully connected graph, a regular lattice, a small world graph and a random graph (see Figure 1). In the baseline regular lattice network each node has \(K = 4\) edges. We further consider extensions to \(K = 6\) and \(K = 8\). All the graphs are connected, that is, there are no nodes that do not have any edges. The fully connected graph is used as a benchmark corresponding to the finite number of agents implementation of the original BH model. Note that in the fully connected graph each node has \(N - 1\) edges which is much larger number relatively to other considered network types. As a baseline model we consider the model with two types of agents, the fundamentalists and the chartists. The simulations are further extended to the model with four types. We analyze the asset price dynamics for \(N = 1000\) agents and, hence, the rewiring probability to obtain the small world network is set to \(\pi = 0.01\). We found that \(N = 1000\) is sufficient for convergence of the discrete choice probabilities (in Eq. 8) to the observed fractions in the case of the fully connected graph.\(^9\) Given that the number of agents in our simulations is finite the system may get locked in the state with only one type. Too avoid this, we introduce two “die-hard” agents who never change their type. They are located on the opposite sides of the network. For comparison we choose the basic parameter values of the model similar to those used in Brock and Hommes (1998), that is, \(r = 0.1, \bar{y} = 10, D = 1, b_f = 0, g_f = 0, c_f = 1, b_c = 0, g_c = 1.2, c_c = 0.\)

\(^8\)The C++ code for our simulations is partially adapted from the code of Bottazzi et al. (2005) and is available on request.

\(^9\)We also analyzed networks with \(N = 100\). Qualitatively the results were similar. However, the level of noise due to the finite sample implementation was much higher.
5.1 Evolution of prices and beliefs

The asset price dynamics for a range of values of $\beta$ are shown by means of bifurcation diagrams in Figure 5. These bifurcation diagrams depict the dependence of the price distribution on the intensity of choice parameter $\beta$. The bifurcation diagram combines two parts: one is initialized at positive deviations from the fundamental price, and the other for the negative deviations of the same magnitude. The price distribution for each level of $\beta$ is represented by a gray-shade histogram. Darker shades correspond to areas of higher density. The histograms are computed using price levels from 10000 periods after 2000 transient periods with $\beta$ ranging from 0.5 to 5 and a linear step of 0.05.

The primary and secondary bifurcations occurring in the fully connected network are similar to the pitchfork and Neimark-Sacker bifurcations occurring in the original BH model for $\beta^* \approx 2.4$ and $\beta^{**} \approx 3.3$ respectively. During the pitchfork bifurcation, the steady state loses its stability and two additional stable steady states are created. The Neimark-Sacker bifurcation leads to the emergence of pe-
periodic or quasi-periodic cycles. The economic intuition behind these bifurcations is as follows. The fundamentalists bring the price to the fundamental level, while the chartists destabilize the fundamental price by extrapolating the trend. The difference in the fractions of these two types determines the price behavior. When the price is close to the fundamental level the excess returns of the fundamentalists and the chartists are equal, but the former incur the costs. When $\beta < \beta^*$ this relative difference in past performance is not important for the choice of the forecasting rule. Thus, the difference in the fractions is not large enough and the price remains at the fundamental level. However, when $\beta^* < \beta < \beta^{**}$, the relative past performance becomes more important and a larger fraction of agents chooses the less costly chartist rule. This results in the deviation of the equilibrium price from the fundamental level. When $\beta > \beta^{**}$, that is, when the agents become highly reactive to the difference in excess returns, we observe cyclical behavior. When the price is near the fundamental level, the fraction of chartists rapidly increases, amplifying any small deviations from the fundamental level and creating a bubble. The bubble ends since the extrapolative behavior of chartists is not strong enough to sustain the trend and at some point fundamentalists start dominating the market bringing the price back to the fundamental level and this sequence recurs.

The values of $\beta$ for which two bifurcations occur in the fully connected network are close to the theoretical values in the BH model. Also for the random graph model the bifurcation values of $\beta$ are close to the values derived for the low dimensional random graph model derived in previous section, namely, $\beta^* \approx 1.07$ and $\beta^{**} \approx 1.35$. The other networks show dynamics consistent with our previous predictions. In particular, in terms of the occurrence of the primary bifurcation with respect to the critical value $\beta^*$ the networks can be arranged in the following order (in decreasing value of $\beta^*$): the fully connected network, the random graph, the small world network, and the regular lattice. The same order holds with respect to the increase
in the price amplitude for fixed $\beta > \beta^*$. These results can be explained by typical characteristics of the network, namely, the size of the neighborhood, or the average degree, $k$, adjusted by its informational content, that is, by the level of independence (disconnectedness) of the neighbors between themselves. The latter is inversely related to the clustering coefficient, $C$. The information about the performance of the alternative type reaches all the nodes in the fully connected network within one time period. As we remove some edges, the neighborhood size decreases and information transmission between the agents who are not directly connected slows down. In addition to the decreased neighborhood size, the informational content of the neighborhood in the regular lattice is further impaired by the direct connections between neighbors (large clustering). In particular, in the neighborhood of size $K = 4$ there are two connected neighbors. This makes the overall speed of the information transmission the slowest for the regular lattice. Slower information transmission results in higher persistence of one particular type over time, or, in other words, it delays the switching. Thus, the fraction of chartists becomes relatively large for smaller values of $\beta$ than in the fully connected network. This translates into earlier bifurcations. The post-bifurcation region of price instability becomes larger and the amplitude of price fluctuations becomes higher. As we start rewiring some of the nodes, the clustering of the network reduces, and the informative content of the neighborhood increases reaching its maximum for the random network.

It is natural to expect that the speed of information transmission may be inversely related to another typical network measure, the characteristic path length measure, $L$. However, the relationship with $L$ is nontrivial, because both $L$ and $C$ are decreasing in rewiring probability $\pi$ for fixed $K$, while $L$ is decreasing in $K$ and $C$ is increasing in $K$ for fixed $\pi$. For fixed $K$ we expect that a shorter characteristic path length, $L$ and, hence, a smaller clustering coefficient, $C$, will result in faster information transmission as observed in Figure 5. To investigate these relationships
Figure 6: Bifurcation diagrams types for varying $k$, $\pi$, $L$ and $C$.

further we plot in Figure 6 the bifurcation diagrams for networks with (a) fixed $L$ and varying $K$, $\pi$ and, hence, $C$ and (b) fixed $C$ and varying $K$, $\pi$ and, hence, $L$. All these networks exhibit small world properties (relatively high $C$ and small $L$). We observe that for fixed $L$, the higher degree, $K$, leads to better information transmission despite the fact that $C$ is increasing with $K$. For fixed $C$, higher $K$ and shorter $L$ leads to improved information transmission as we would expect. Hence, we conclude that $L$ and/or $C$ by themselves are not sufficient measures to represent the speed of information transmission in the BH type model with a network. As we previously discussed a “clustering-adjusted” degree could be an appropriate measure for our set-up. However, the exact definition of this measure is left for future work.

Figure 7 depicts the time series of the price for two values of the intensity of the switching parameter, $\beta = 1$ and $\beta = 3.5$, and the four networks: the fully connected graph (FC), the regular lattice (RL), the small world network (SW) and the random graph (RG). We use this abbreviation in subsequent figures. For $\beta = 1$ the price dynamics corresponding to the fully connected graph and the random network converges to a steady state, while the regular lattice and the small world network lead to irregular asset price fluctuations. For $\beta = 3.5$, somewhat regular fluctuations
Figure 7: Time series of price.

emerge for all the network topologies, however, the regularity and the amplitudes of fluctuations vary considerably among them. The price dynamics of the random network are close to those of the fully connected network. The price dynamics produced by the regular lattice are the most distinct from the fully connected network. The small world network produces price dynamics similar to the regular lattice with some shift towards the random graph. The observed price behavior is indeed consistent with the previously deduced bifurcation values of $\beta$ for different network types.

To provide insights into the effects of different network topologies on market behavior we track how individual agents change their forecasting beliefs over time. Figure 8 shows a typical set of patterns that emerge during simulations. This set is for $\beta = 3.5$. The figure shows the evolution of the forecasting type for all 1000
agents at every time step from 0 to 1000. Each point on a vertical line represents an agents’ type: a black point indicates the fundamentalist type, while a blank (or white) point indicates the chartist type. In terms of the spacial configuration, the agents are numbered in the natural order for the regular lattice and each vertical line corresponds to the circular network representation depicted in Figure 1. The circle is broken between agent 0 and agent 1000 to be represented as a line. The inner-circle connections are not explicitly shown on the line, but the network configuration can be deduced from the time-evolution of agents’ types. Agent 0 is a “die-hard” chartist and agent 500 is a “die-hard” fundamentalist. These two agents never change their type.

The periods of the highest concentration of fundamentalists correspond to the time when the price falls to the fundamental level, while the lowest concentration of fundamentalists corresponds to the highest deviation from the fundamental value of the price. Since the bifurcation values of $\beta$ depend on the network type, the four models may be at different stages of development for fixed $\beta = 3.5$. Hence, the direct comparison of the models is not formally possible. For each network type, however, the observed patterns are somewhat representative of the behavior subsequent to the secondary bifurcation. They can be used to better understand price dynamics on the right side in Figure 7. Overall, the fraction of fundamentalists is relatively
high in the fully connected network. This is consistent with smaller deviations from the fundamental price and frequent price oscillations. Fundamentalists are relatively uniformly distributed across the network. Large spikes in the fractions of the fundamentalists correspond to price falls. In the case of the regular lattice we observe high clustering of the fundamentalists around the fundamental “core”. This is consistent with the high clustering coefficient of this network. In the small world network we also observe clusters, but they are smaller and more disperse in space. Again this is consistent with sparsity and a high clustering coefficient typical for this network. In the case of random graph we do not observe any clusters of the fundamentalists. This is due to a very small clustering coefficient for this network and a relatively small number of the fundamentalists in the market during most of the periods.

Figure 9 presents time series plots for the asset price corresponding to the case of stochastic dividends. We assume that the dividends are independent and identically normally distributed with the mean set to 10 and the variance of 1. For $\beta = 1$, even with the small variance in the dividend process, the time series of the small world network and the regular lattice exhibit large fluctuations. For the fully connected network and the random graph the price stays at the fundamental level of 100. For $\beta = 3.5$, the regular lattice and the small world network produce even greater irregular single-peak deviations from the fundamental price, while the fully connected and the random networks produce relatively regular fluctuations of a much smaller scale. A comparison of Figure 7 with Figure 9 suggests that the impact of the stochastic dividend on the price is the strongest in the small world and the regular lattice networks.

The informational efficiency is closely related to the speed of information transmission and can be measured by comparing the volatility of the observed price with the volatility of the fundamental dividend process as suggested by Shiller (1981). In
order to abstract from the effect of the time-varying dividend in our model, we keep the dividend process constant. Under this assumption, the Efficient Market Hypothesis would predict constant price over time and zero trading volume. In Figure 10 we analyze the standard deviation of the price (panel a) and the average traded volume (panel b) for values of $\beta$ ranging from 0.5 to 5 for the four topologies. We ignore the first 2000 transitory iterations and compute the standard deviation of the price and the average traded volume for the following 2000 periods. To eliminate the dependence of our results on a particular realization of the random seed, we report averages for 100 simulation runs, each run having its own random seed. The same simulation setup is used for all the other statistics reported further. We observe that the random graph and the fully connected network exhibit the most informational
efficient outcomes for any values of $\beta$ which is consistent with the highest speed of information transmission in these two networks. The regular lattice exhibits the least informationally efficient outcome.

### 5.2 Statistical properties

Below we analyze the properties of the time series generated by the four considered networks. We attempt to relate these properties to the stylized facts of the financial returns.

Figure 11 depicts the skewness of the returns and the kurtosis of the returns. The former statistic (Figure 11a) measures the asymmetry of the distribution. It is
close to zero for all the networks for all post-bifurcation values of $\beta$. The returns generated by the model with the small world network are slightly negatively skewed.

The kurtosis plot (Figure 11b) reveals that all the four networks generate return distributions with different kurtosis values. The small world network return distribution exhibits the kurtosis value around 8, which is relatively close to the one observed for the returns on the financial markets.

By computing the autocorrelation of the returns for the four network structures, we can analyze linear unpredictability of the stock returns, which is a well-known property of the time series exhibited by the real financial markets. Figure 12a depicts the autocorrelation of returns for the first five lags as a function of the intensity of choice. Usually the real financial time series exhibit small or no autocorrelation of returns. The regular lattice and the small world network produce high autocorrelations at all lags. This, again, can be attributed to a less efficient information transmission in these networks. Although the random graph and the fully connected network display large autocorrelations at the first two lags, they converge to zero autocorrelation values at lag three to five. The significant positive autocorrelations are resulting predominantly from the persistence of chartists strategies. It is possible to reduce the autocorrelations by adding a sufficient amount of dynamic noise into the price as in Hommes (2002). However, we do not aim to reproduce stylized facts in this paper and therefore do not pursue this route.

Figure 12b shows the correlations between the squared returns and the volume of trades. In real financial markets, high trade volumes are associated with high volatility. Many standard asset pricing models, however, fail to reproduce this relation. Our model produces positive volume-volatility correlations for all networks. The highest values in the post-bifurcation region are observed under the random graph network, followed by the small world network.

A universal property of the real financial time series is the volatility clustering,
Figure 12: Properties of returns and volume.

Figure 13: Autocorrelation of squared returns.

i.e., the presence of slow decaying autocorrelations in the squared returns. Figure 13a shows the autocorrelations of the squared returns at the first five lags as a function of $\beta$, while Figure 13b shows the autocorrelation function of the squared returns for 20 lags with $\beta = 3.5$. The autocorrelations of the squared returns under the fully connected network and random graph vanish after the first few lags, which is not consistent with stylized facts. In turn the autocorrelations under the regular lattice and the small world network remain positive and large at many lags for the regular lattice and the small world network, indicating the volatility clustering of the returns.

The above analysis reveals that different local interaction arrangements in the
market affect the dynamics and the time series properties. The effect of the change in the behavior parameter $\beta$ also depends on a particular network configuration.

5.3 Robustness to the ecologies with more belief types

In this section we extend the baseline BH model with fundamentalists and chartists to the model with four belief types by introducing two additional types, i.e., positively biased extrapolators and negatively biased extrapolators (see section 4.3 in Brock and Hommes, 1998). This model is chosen because for some parametrization it exhibits almost no autocorrelations in the chaotic return series (Hommes, 2006). The general form of all belief types is given in Eq. 5. The parameters used for the fundamentalists and chartists are set to $b^f = 0, g^f = 0$ and $b^c = 0, g^c = 1.21$, respectively, and the parameters for the two new types, the positively and negatively biased extrapolators, are set to $b^p = 0.2, g^p = 1.1$ and $b^n = -0.2, g^n = 0.9$; cost $c$ were set to 0 for all types, $r = 0.1, \bar{y} = 10$. The number of agents is set to $N = 1000$ and similarly to the two-type case, the network is populated by four equidistant “die-hard” agents who never change their types. As we have mentioned in Section 4.2, in principle, it is possible to derive a low dimensional approximation for the random network model with four types, but these derivations would be rather involved and we leave this for future work. Instead, we resort directly to simulations.

Figure 14 shows the bifurcation diagram for the model with four belief types. For the fully connected graph, the bifurcation seems to occur around the value of $\beta^* = 50$. This is the value for which the Neimark-Sacker bifurcation occurs in the BH model with four belief types. For large values of $\beta > 85$ chaotic behavior can be inferred in the BH model. This coincides with the region for $\beta$, where we observe the

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10This is a standard specification used in Brock and Hommes (1998). We also have tried an alternative specification in Hommes (2006), but the results did not change qualitatively.

11Given that we have 4 types now, there are several possibilities on how to locate "die-hard" agents relative to each other. We have tried various permutations, but they did not influence the results.
highest price fluctuations in the case of the fully connected network. In the case of
the random network the bifurcation seems to be occurring for the values just before
theoretical $\beta^* = 50$ and the observed price fluctuations are higher relatively to the
fully connected network. These results are somewhat similar to what we have ob-
served in the two-type model. Contrary to the case of the two-type model, we do not
observe large price fluctuations or clearly defined bifurcations for the regular lattice
and the small world networks. To shed some light on why this might be happening
we analyze the behavior of the four-type BH model with asynchronous updating
(see Section 4.2 for detailed discussion on how the asynchronous updating is related
to the local information exchange). Multiple bifurcation diagrams constructed in
the E&F Chaos\footnote{Not provided here for brevity, but available on request.} indicate that the critical bifurcation value, $\beta^*$, depends on the
fraction of agents who keep (do not update) their beliefs. As this fraction increases
the value of $\beta^*$ increases. For large fractions of non-updating agents, the ampli-
tude of price fluctuations is rather small. These findings indicate that the observed
behavior for the regular lattice and the small world may be anticipated to some

Figure 14: Bifurcation diagrams for four-type models.
degree. Figure 15 shows a typical time series for the four-type models for different values of $\beta$. For small $\beta = 30$ we observe small amplitude noise fluctuations, closely resembling random walk series for the regular lattice and the small world networks. For larger $\beta = 85$, the price deviations for the regular lattice and small world exhibit more regularly and some volatility clustering. The fully connected graph shows symmetric fluctuations of some regularity, while the random graph shows asymmetric fluctuation with large positive price spikes. Given that bifurcations seem to occur at different values for different networks, care needs to be exercised in comparing these results.

We considered other network topologies with the degree $K = 6$ and $K = 8$ but the results were qualitatively similar. In particular, as the degree increases,
Table 2: Characteristics depending on the network in increasing order left to right.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Two-type model</th>
<th>Four-type model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latency in information transmission</td>
<td>FC RG SW RL</td>
<td>FC RG SW RL</td>
</tr>
<tr>
<td>1/β of the primary bifurcation</td>
<td>FC RG SW RL</td>
<td>FC RG</td>
</tr>
<tr>
<td>Amplitude</td>
<td>FC RG SW RL</td>
<td>RL SW FC RG</td>
</tr>
<tr>
<td>Irregularity of dynamics</td>
<td>FC RG SW RL</td>
<td>FC RG SW RL</td>
</tr>
<tr>
<td>Std. deviation of price</td>
<td>FC RG SW RL</td>
<td>RL SW FC RG</td>
</tr>
<tr>
<td>Average trading volume</td>
<td>FC RG SW RL</td>
<td>RL SW FC RG</td>
</tr>
<tr>
<td>Skewness of returns</td>
<td>SW RG FC RG</td>
<td>RL SW FC RG</td>
</tr>
<tr>
<td>Kurtosis of returns</td>
<td>RG SW RL FC</td>
<td>SW RL FC RG</td>
</tr>
<tr>
<td>Autocorrelations of returns</td>
<td>FC RG SW RL</td>
<td>inconclusive</td>
</tr>
<tr>
<td>Volume/volatility correlations</td>
<td>FC RL SW RG</td>
<td>FC (neg) SM RL RG</td>
</tr>
<tr>
<td>Autocorrelations of squared returns</td>
<td>FC RG SW RL</td>
<td>FC RG RL SW</td>
</tr>
</tbody>
</table>

the dynamics in the small world case greater resembles the observed random graph dynamics (when we hold clustering coefficient C fixed). We further qualitatively compare various characteristics of the two-type and four-type models for the baseline networks with $K = 4$ (see Table 2, increasing order in terms of the network from left to right). First, we note that the ordering of the properties is rather different for the two-type and four-type models. We also note that in the two-type model the ordering of the statistical properties of returns, except skewness and volume/volatility correlations, is consistent with the ordering of latency in the information transmission. This ordering is not clearly defined for the four-type model.

6 Conclusions and extensions

In this paper we expanded the model of Brock and Hommes (1998) by introducing local information exchange via communication networks. We studied how different network structures affect asset price dynamics. We derived a low dimensional system to represent dynamics in the two-type model with random graph and discussed stability for this case. Other network structures were investigated by simulations.
We observed that the stability regions with respect to the intensity of choice parameter $\beta$ depend on the parameters of the communication network. For the two-type model a relatively slower information transmission in the regular lattice and the small world networks creates greater information inefficiencies and induces greater instabilities and higher deviations in the price dynamics. The speed of the information transmission is increasing in the (average) degree of the network and decreasing in its clustering.

The work in this paper may be extended in a number of directions. (1) The basic principles used to derive the low-dimensional analytically tractable model for the random network may be used to derive similar models for small world networks. Moreover, it would be interesting to extend this work to the scale free networks and other topologies popular in the literature. Other agent ecologies may also be extended to incorporate various network structure. (2) Another interesting direction is to make the strength of the noise, $1/\beta$, dependent on the number of neighbors of own and alternative types. This to some degree would endogenize parameter $\beta$. (3) In addition to this, it would be important to consider agents with longer memory who would consider the strategies they used more than one period ago or perform some counterfactual analysis if the performance of an alternative type is not observed. (4) In many real-life networks there is a feedback between network performance and network formation. The performance of agents may gradually influence the network topology they are active in. Extending the model to include endogenous network formation would also be of great interest.

References


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