The Changing International Network of Sovereign Debt and Financial Institutions

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Abstract
We develop a theoretical and empirical framework for the connections between global financial and sovereign CDS markets. The transmission of shocks is shown to affect the systemic default probability of the international network. The network is found to be "robust but fragile", meaning that a shock can result in the propagation of crises. Between 2003 and 2013, the probability of default in the network in the face of potentially poor investment outcomes and/or sovereign bond haircuts changes substantially. The results suggest that it is the interconnectedness of the financial and sovereign debt markets that provides increased protection against financial fragility.

Keywords: network, sovereign debt, financial institutions, systemic risk, contagion

JEL classification: G01, C58, C31

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Disclosure Statement

Mardi Dungey declares that she has no relevant or material financial interests that relate to the research described in this paper.

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1 Introduction

International connections between banks and sovereign debt markets are important for financial and economic stability. However, the complex nature of these connections and the direction of influence remains a matter of debate; see, for example, discussions in Reinhart and Rogoff (2009) and Kallestrup et al (2016).\footnote{Other relevant literature includes Kalbaska and Gatowski (2012), Alter and Schuler (2012), Ureche-Rangan and Burietz (2013), Bruyckere et al (2013).} Theoretical arguments can be made for poor macroeconomic policy weakening the state of the economy and transferring stress to the banking sector. Alternatively, an abrupt interruption of private credit growth may weaken the banking market to the extent that it requires government support, in turn undermining sovereign creditworthiness. A spiral of weakening banks and weakening sovereign credit may result when banks have significant holdings of sovereign debt. These network effects can spill over to the real economy; see Carvahalo (2010), Acemoglu et al (2012), Anufriev and Panchenko (2015).

This paper develops a theoretical and empirical framework for analyzing the network of connections between financial institutions and sovereign debt, focusing on evidence of changes in the network structure during periods of market stress. The framework provides a mechanism by which ‘robust-but-fragile’ networks may emerge in the face of an unexpected shock to the system through poor investments and/or poor government policies.

Our theoretical framework consists of two sectors: financial institutions (banks) and sovereigns. Banks are engaged in capital lending with one another without a requirement to borrow all available funds. Each bank has an opportunity to invest in the real economy with an uncertain return. Financial institutions that choose to retain (some) funds in the sovereign debt market face the risk of a haircut. In this case, both sectors experience a potential underinvestment problem caused by an external shock. The volatility of this shock has nonlinear effects on the network, and illustrates how the default of a financial institution and/or sovereign can potentially increase the probability of other
entities defaulting.

By generalizing the results of Acemoglu et al (2015), we show that the default of a sovereign or financial institution as a result of the unexpected shock leads to multiple equilibria defined by a collection of mutually consistent payments between entities. To analyze the continuum of equilibria, we show how to assess the sensitivity of the counterparty to default. This quantity measures how counterparty default influences the ability of an entity to meet obligations.

Given the existence of multiple equilibria, systemic default probabilities occur in different scenarios related to normal or poor investment outcomes. In particular, we consider the following scenarios: Good times, in which returns in the real economy are good and there are no sovereign defaults, in which case the network is relatively robust; Poor investment, in which returns in the real economy are poor but there are no sovereign defaults; Poor government, in which returns in the real economy are good but government policy leads to sovereign bond haircuts; and Stress conditions, in which both poor returns in the real economy and sovereign haircuts stress the network. These scenarios permit identification of the efficient equilibrium as the state that minimizes the aggregate loss of all creditors.

Empirically, we provide evidence on three specific hypotheses regarding the changing nature of a global network comprising 67 financial sector institutions and 40 sovereigns via the CDS market over the period 2003 - 2013. Specifically, our framework provides evidence for (i) changes in link strength between nodes, consistent with existing tests of contagion; (ii) changes in the number of connections between nodes, that is, the network may become more or less dense, as evidenced in existing network literature; and (iii) changes in network completeness weighted by the strength of linkages, combining information on the existence of linkages and their relative importance. We test for potential changes in the network from September 15, 2008, consistent with the global financial crisis initiated by the collapse of Lehman Brothers, and from April 1, 2010, consistent with the period of the Greek and subsequent sovereign debt

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2The first two cases are consistent with the original Acemoglu et al (2015) analysis.
crises in Europe.

Our evidence supports a high degree of network completeness, consistent with the major role played by common factors in Longstaff et al (2011). The empirical framework is based on a network of edges assessed by Granger causality tests; see also Billio et al (2012) and Merton et al (2013). Drawing on Diebold and Yilmaz (2009, 2014, 2016) and weighting the existence of linkages by their strength, we show that financial network completeness falls; see also Atil et al (2016) and Fabozzi et al (2016). The number of links and the completeness of the network incorporating financial institutions and sovereigns may increase across certain crises, as in Billio et al (2012), or fall, as in Caporin et al (2014). Net declines in network completeness may, for example, represent the removal of a large number of weaker linkages and their replacement with a small number of stronger linkages, resulting in differences in network topology between periods. This is important because policymakers may wish to react quite differently to a larger, more loosely connected network than to a more concentrated strongly connected one; see, for example, the literature on bank concentration during crises, Beck et al (2006).

Our approach nests tests for contagion in a systemic risk assessment through the removal and formation of new linkages during periods of financial stress. Contagion is defined as the formation of new linkages, such as new commonalities between formerly unrelated assets, see Bekaert et al (2014) and Dungey and Martin (2007), or the breakdown of linkages between counterparties (Gai and Kapadia, 2010) and differences in the transmission mechanisms for tail-shocks, as in Boyson et al (2010) and Busetti and Harvey (2011). By using the Granger causality framework, we are methodologically associated with the contagion literature; see Longstaff (2010), Marais and Bates (2006) and Sander and Kleimeier (2003).\(^3\)

\(^3\)Acemoglu et al (2015) use the term contagion to denote the transmission of shocks across their networks based on the known lending relationships between banks. Their usage is more consistent with spillovers, where spillovers are ex ante known linkages between nodes; contagion is usually used to refer to transmission of shocks beyond that indicated by the usual linkages. For an overview, see Dungey et al (2005).
Evidence of changes in the network structure around the timing of the global financial crisis supports shifts in relationships between financial institutions and sovereign debt markets. One form of these shifts is consistent with both a global flight from markets with heavily increased risk during the crisis – notably source markets from European sovereigns and US financial institutions – represented by the breakdown of network linkages. The other form represents seeking new markets, consistent with a shift in relative risk/return trade-offs globally – notably increased linkages with Africa – represented by the formation of new links in the network.

We calculate the expected number of defaults for the network in different sample phases in four different scenarios (Good times, Poor investment, Poor government and Stress) to illustrate the difficulties inherent in financial regulation. In the period prior to the Lehman Brothers collapse, the expected number of defaults for all entities in the Poor government and Good times scenarios is relatively low. In this case, even a large shock is expected to cause only a small number of defaults. The combined network is more fragile in the Poor investment scenario. However, in crisis periods, the expected number of defaults in the combined network is almost the same in the Poor government and Poor investment scenarios, indicating feedback effects between banks and sovereigns. Default risk is highest in the second phase, equivalent to the global financial crisis period, when it is almost five-fold that of the pre-crisis period. During the period of the Greek and European sovereign debt crises, the third phase, default risk returns to previous levels in the case of Poor government, while under Stress conditions and Poor investment, the expected number of defaults is lower than before. An immediate practical outcome of these results is to support the calls for regulators to recognize the need for non-zero risk weightings on sovereign debt in bank capital assessment; see Hannoun (2011) and, for evidence on the potential importance of the so-called zero risk practice, Korte and Steffen (2015).

The paper proceeds as follows. Section 2 discusses the theoretical framework for modelling a network incorporating financial institutions (banks), real econ-
omy firms and sovereign debt. Section 3 explores the data set used for empirical analysis. The econometric methodology for establishing edges is outlined in Section 4, and Section 5 presents results for our sample of 107 entities (financial institutions and sovereigns). Section 6 concludes.

2 Theoretical Framework

Papers by Gai and Kapadia (2010) and Acemoglu et al (2015) propose a theoretical framework for modeling networks between banks and simulate the transmission of shocks through banking networks. We extend the framework of Acemoglu et al (2015) to consider that banking network connections are susceptible to potential defaults on sovereign debt. Acemoglu et al (2015) specifically include the possibility that banks do not take full advantage of all possible borrowing options available within their bank counterparty network and instead ‘hoard funds’ as a form of cash reserve. In particular, they identify this component of a bank’s portfolio decision as representing investment in sovereign debt, which provides a standard riskless return, $R$, thus acting as cash in their model. Our extension speaks strongly to outcomes for banking networks when the return on sovereign debt is uncertain. Specifically, we show that the results in Acemoglu et al (2015) regarding the stability and fragility of banking networks transfer directly to the extended network between sovereign debt markets and banking. This combined network is highly interconnected, demonstrating a robust-yet-fragile structure, Haldane (2009), which is at risk when exposed to a large enough single shock, or sufficiently proximate contemporaneous small shocks.

2.1 The model of banks and sovereigns

Banks are in the business of lending for projects with uncertain returns. As in Diamond (1982) banks cannot fund their lending activities from their own balance sheets and need to engage in inter-bank relationships. This creates networks of liabilities between banks, where the edges are determined by repayments required between pairs of financial institutions. Banking networks
become highly interconnected as banks can hold assets and liabilities with any number of other banks in the network, as in Allen and Babus (2009).

Consider a risk-neutral bank, operating in a three-period time frame as in Acemoglu et al (2015). Each bank has capital to lend and a pre-determined individual credit limit with each other bank in the network (total available loans to bank $i$ from bank $j$ may not match the corresponding figure available to bank $j$ from bank $i$). No bank is required to borrow the total credit available, but they are all required to settle all interbank liabilities at each $t = 1, 2$.

Each bank has the opportunity in period $t = 0$ to pursue an investment opportunity in the real economy, $z_i$, with an uncertain return in period $t = 1$ but a certain, non-pledgable return at $t = 2$. If the bank needs to redeem its investment at $t = 1$ the fraction of the loan it may recover is assumed to be small.

Our extension concerns funds that banks choose not to invest. In Acemoglu et al (2015) these funds, denoted $c_i$, are considered equivalent to a sovereign bond, bearing a certain return, $R$, with no risk. We extend the analysis to consider not only the investment project having an uncertain payoff but also a haircut risk to the sovereign bond. In this case, in period $t = 1$, the values of returns $z_i$ and $c_i$ are influenced by an external shock, $u_i$, which is a random variable drawn from a given distribution with mean zero and variance one, and its standard deviation is $\sigma_i$. The joint probability distribution $p(u_1, ..., u_n)$ for $n$ entities is assumed to be known.

**Definition 1** Network $G$ is the pair $(N, E)$, where $N$ is a set of nodes representing entities (banks or sovereigns), and a set of edges $E$ represents contracts between two entities from lender to borrower.

After making investment decisions in the initial period, at time $t = 1$, the

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4 Shock $u_i$ contains uncertainty about sovereigns and financial institutions and can be seen as an aggregated shock. However, it is trivial to separately analyse disaggregated shocks. Acemoglu et al (2015) and Glasserman and Young (2015) imply that shocks have a negative impact on returns. In this paper, the shock $u_i$, takes a value between -1 and 1, which permits positive shocks.

5 In the following discussion, the term entity means bank (financial institution) or sovereign.
bank has to settle counterparty liabilities with other banks in the network, \( y_j \), and meet its unavoidable contractual obligations, such as wages, and required payments to investment projects, \( v_j \). We assume that \( v_j \) is senior to counterparty payments. Resources available to make these repayments, \( \alpha_j \), consist of the funds the bank placed in sovereign bond investments, \( c_j \), uncertain first-period returns on the investment project, \( z_j \), and repayments from counterparties, \( \sum_{i \neq j} x_{j,i} \).

Define a default indicator \( d_j \), with \( d_j = 1 \) if entity \( j \) defaults at time \( t = 1 \) and \( d_j = 0 \) otherwise. If all entities are assumed to default, \( x_{j,i}(d_j) = 1 \), \( \forall i, j \). An entity \( j \) defaults if its assets, \( \alpha_j \), at time \( t = 1 \) are smaller than total liabilities \( l_j = v_j + y_j \). That is the entity will not be able to meet its first-period obligations in the case in which

\[
\alpha_j = c_j + z_j + \sigma_j u_j + \sum_{i \neq j} x_{j,i} < v_j + y_j. \tag{1}
\]

The default condition, defined in equation (1), is expressed as a function of the stochastic shock, \( u_j \), that impacts both sovereign and bank returns\(^6\), which permits us to rewrite (1) as

\[
u_j < \frac{c_j + z_j + \sum_{i \neq j} x_{i,i} - v_j - y_j}{-\sigma_j} = q_j, \tag{2}
\]

in which \( q_j \) defines the threshold value for shock \( u_j \). If \( u_j < q_j \), the shock causes the default of entity \( j \). In this case, sign and magnitude of shock \( u_j \) impact the default probability of entity \( j \) and its obligations, potentially increasing the default probability of other entities. Moreover, the default probability also depends on the volatility of shock \( \sigma_j \) that nonlinearly affects the network. It is unlikely that an absolute threshold value can be established across multiple markets (for example, banking sectors of individual economies), as the variability of shocks may vary across different networks.

Equation (2) can be used to express the default condition of \( n \) entities in

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\(^6\)This implication is different from Acemoglu et al (2015), who assume that \( c_j \) is certain, which means that risk arises entirely from the uncertainty affecting \( z_j \). Note that \( u_j \) can be easily expressed as a combination of independent shocks affecting the different entities.
terms of the default indicators, $d_j$, as

$$d_j = 1(q_j - u_j),$$

(3)
in which 1 denotes the step function, which is equal to one if the argument is positive and zero otherwise\(^7\). A solution of the system (3) is represented by vector $\hat{d} = (d_1, ..., d_n)$.

**Definition 2** An equilibrium is defined by a vector of default indicators, $d = (d_1, ..., d_n)$, that is a solution of equation (3).

In general, a solution of equation (3) can be represented by multiple equilibria, which have two sources. First, an interdependence of entity liabilities might imply more than one vector $\hat{d}$. Second, there can be multiple values of shocks $u_j$ that solve equation (3). While a typical approach in the literature\(^8\) is to focus on the best-case equilibria, in which as few entities as possible default, we show in the following section that a multiplicity of equilibria can be useful for analyzing different scenarios of the changing international network.

As every counterparty can have only two states (default, $d_j = 1$, or no default, $d_j = 0$). Definition 2 implies that $q_j$ for every entity $j$ can take a finite number of values, $2^{n_j}$, where $n_j$ is the number of counterparties borrowing from entity $j$. Given that shock $u_j$ is a random variable taking values on the interval $[-1, 1]$, it is important to identify conditions under which the value of $u_j$ has an impact on the default of entity $j$.

**Proposition 1** Under the mapping $\tilde{q}_j = \min\{\max\{q_j, -1\}, 1\}$ the default is a function of shock $u_j$.

Proposition 1 implies that when $-1 < q_j < 1$, shock $u_j$ affects network $G$. It unnecessary to constrain the values of $q_j$; however, this range is of interest in default scenarios, a primary focus of this paper. When the values of $q_j$ lie outside $[-1, 1]$, the results become independent of shock $u_j$.

\(^7\)This notion is similar to Roukny, Battiston and Stiglitz (2016). An alternative formulation of the default condition counts losses in dollars related to the insolvency of entities of the network rather than the number of defaults (Glasserman and Yong, 2016), which is consistent with the SRISK approach promoted by the NYU Stern Volatility Laboratory project.

\(^8\)see, e.g., Elliott et al. (2014)
2.2 Multiple equilibria

The main task now is to identify how the network structure leads to unique or multiple solutions of system (3). Following default condition (2), a collection of mutually consistent payments between entities at \( t = 1 \) define the network structure, also defined by an entity’s borrowing counterparties’ default state. This setting implies that a shock to an entity might not only lead to that entity’s default, but might also initiate a cascade of creditor defaults, permitting financial contagion in the system.

Definition 3 A walk \( P_{j_1,j_k} \) is a sequence of entities \((j_1, \ldots, j_k)\) such that the pairs \((j_1, j_2), (j_2, j_3), \ldots, (j_{k-1}, j_k)\) \( \in E \) are edges of the network. A walk is closed if the first and the last institution in the sequence are the same, and open if they are different. The length of the walk \( P_{j_1,j_k} \) is given by the number of edges, \( k \), contained in it. A cycle \( C_n \) is a closed walk represented by \( n \) entities and \( n - 1 \) edges.

Following Definition 3, in a network of banks and sovereigns, a cycle is an arrangement of contracts that can be represented by a circle in which an entity \( j \) borrows from one neighbor and lends to another neighbor.

Definition 4 The sensitivity of entity \( j \) to the default of counterparty \( i \) is

\[
scd_{j,i} = \tilde{q}_j(H^k_{i=1}) - \tilde{q}_j(H^k_{i=0}), \forall i \neq j
\]

in which \( H^k_{i=1} \) is the realization of defaults of \( k \) entities subject to \( d_i = 1 \).

Entity \( j \) may also fail if counterparties fail to pay their debts, providing another source of multiple equilibria. Inability to pay is closely related to the sensitivity of the counterparties of entity \( j \), as defined in (4). In particular, the sensitivity of counterparties to default measures how the inability of entity \( j \) to meet its obligations influences entity \( i \), which implies that more fragile entities have a higher sensitivity to default. The presence of cycles in the network and dependencies between the liabilities of entities are important conditions for generating multiple equilibria, as formalized in the following proposition.
Proposition 2 Consider a network of $n$ institutions with $x^d_j = \sum_{i \neq j} x^d_{j,i}$, in which $x^d_{j,i}$ is the amount of money that institution $j$ recovers from the default of institution $i$. Suppose that $x^d_j < 1$, unavoidable contractual obligations $v_j > 0$, and the shock variance $\sigma_j > 0$ is finite. Multiple equilibria exist if and only if

(a) a cycle $C_k$ of contracts along $k \geq 2$ institutions exists;

(b) for each entity in cycle $C_k$, the sensitivity to default $scd_{j,i} \neq 0$.

Necessary condition (a) in proposition 2 relates the existence of multiple equilibria to the network structure. In particular, the network contains at least two entities with interdependent default conditions. Sufficient condition$^9$ (b) specifies how defaults influence entities belonging to cycle $C_k$. If for a given shock $u_j$, the default condition of institution $j$ does not change, regardless of whether the borrowing counterparties of $j$ default, there are no multiple equilibria for institution $j$. Formally, realizations $\mathcal{H}^d_j$ are identical for $j = 1$ and $j = 0$.

An important implication of proposition 2 is that financial network equilibria are defined by the default indicators, $d_j$, dependent on shock $u_j$. In fact, condition (b) implies there is a large shock, $u_j^*$, triggering institutions in cycle $C_k$. This finding motivates incorporating unexpected shocks into the empirical framework, as proposed in the following section. Variance decompositions capture the impacts of shocks on the network, relating our methodological framework to the approach of Diebold and Yilmaz (2014). Proposition 2 also suggests that an acyclical network, such as a tree, will have a unique equilibrium for all possible realizations of shocks. Moreover, if the nodes of a network are represented by only outgoing or incoming links, there is only one equilibrium. In other words, if entities either only borrow from or only lend money to their counterparties, a continuum of equilibria does not exist; networks need at least one bivariate linkage to generate multiple equilibria in this framework.

$^9$If shock $u_j = 0$ for all institutions, the sufficient condition (b) is similar to the unique payment equilibrium condition of Acemoglu et al (2015), which is $\sum_j (z_j + c_j) \neq nv$. This condition restrict banks to default due to "coordination failures".
Definition 5 Given a distribution $p(u)$, the expected number of defaults is represented by $\sum_{i=1}^{n} scd_{j,i}^2/n$.

As follows from Definition 5, the expected number of defaults in the network is defined by sensitivity indicators $scd_{j,i}$ aggregated across all entities. This measure depends not only on the size of the realized shocks, $u_j$, but also on the variance of shocks, $\sigma_j$, and the network structure. Moreover, the expected number of defaults measure is closely related to a variance decomposition. This result motivates the proposed econometric framework, which will be discussed in greater detail in the following sections.

2.3 Systemic default probability

Given the solution of system (3) and a joint probability density function of shocks $p(u)$, following Roukny et al (2016), a systemic default indicator $d^{sys}$ can be defined as

$$d^{sys} = 1_{\{\sum_j d_j \geq n^*\}},$$

(5)

in which threshold $n^*$ defines how many entities initiate systemic default. If $n = n^*$, systemic default is a situation in which all entities cannot meet their obligations. If there is a unique equilibrium for the default state, $d$, the systemic default probability can be defined as

$$P^{sys} = \int d^{sys}(u)p(u)du,$$

(6)

in which $d^{sys}$ is estimated from equation (5), where $d$ is a solution of equation (3). Note that any possible correlation structure across shocks can be incorporated in $p(u)$.

In the case of multiple equilibria, the systemic default probability cannot be estimated directly from equation (6), as $d^{sys}(u)$ can take several values. However, for a given shock, multiple equilibria can be analyzed according to different scenarios related to different values of $q_j$ defined in equation (2).

Different states of nature are related to each of the project investments, $z_j$, and the sovereigns, $c_j$. In good (or normal) times, banks and sovereigns will
achieve standard returns $z_j^+$ and $c_j^+$, respectively; in a poor outcome period they are subject to a haircut, receiving $z_j^-$ and $c_j^-$. In this case, there are four separate potential scenarios that the entity network may face at $t = 1$ to meet its liabilities.

**Good times:** Investments achieve payoff $z_j^+$, and there are no haircuts in sovereign debt markets, which is equivalent to the good case in Acemoglu et al (2015). Bank networks should function normally - all sources of income are available to meet liabilities.

$$c_j^++z_j^+ + \sum_{i\neq j} x_{j,i} \geq v_j + y_j.$$ 

**Stress:** Investments do not perform, providing payoff $z_j^-$, and there is poor performance in the sovereign debt market that necessitates haircuts, which provides a sovereign with negative return $c_j^-$. An entity’s incoming counterparty payments need to exceed outside obligations owing due to the investment, the entity’s own outgoing counterparty requirements, investment losses and the haircut.

$$\sum_{i\neq j} x_{j,i} > v_j + y_j - c_j^- - z_j^-.$$ 

**Poor investment:** Investments do not perform, and a bank receives payoff $z_j^-$. However, sovereign debt markets perform normally with no haircuts required, which yields $c_j^+$. Bank bond holdings and incoming counterparty payments need to exceed outside obligations owed, the bank’s own owed counterparty requirements and the loss due to the poor investment outcome. This is equivalent to the bad case in Acemoglu et al (2015).

$$c_j^+ + \sum_{i\neq j} x_{j,i} > v_j + y_j - z_j^-.$$ 

**Poor government:** Investments perform well and provide payoff $z_j^+$. However, there is poor performance in the sovereign debt market entailing negative returns, $c_j^-$. An entity’s income from successful investments and incoming coun-
Interparty payments need to exceed outside obligations due to the investment, the entity’s own outgoing counterparty requirements and the haircut.

\[ z_j^+ + \sum_{i \neq j} x_{j,i} > v_j + y_j - c_j^- . \]

These scenarios permit the estimation of the systemic default probability in the case of multiple equilibria. Consider the set \( \Omega \supseteq \{ I_i \cup I_g \cup I_s \} \) of all possible solutions of equation (3) for a given shock \( u \), in which \( I_i \) is a set of equilibria in the poor investment case, \( I_g \) is a similar set for the poor government case, and \( I_s \) is assigned to the stress scenario. Moreover, for set \( \Omega \), the best-case equilibrium can be defined as \( d_{i\text{sys}}(u) = \inf_{k \in \Omega} d_{k\text{sys}}(u) \), while the worst equilibrium is \( d_{\text{sys}}(u) = \sup_{k \in \Omega} d_{k\text{sys}}(u) \). These definitions select the best (worst) solutions according to smallest (largest) number of defaults in the entity network. The systemic default probability in the worst and best scenarios, respectively, can be defined as

\[ P_{\text{sys}}^w = \int d_{i\text{sys}}(u)p(u)du, \]

\[ P_{\text{sys}}^b = \int d_{\text{sys}}(u)p(u)du. \]

Note that if maximal and minimal values \( P_{\text{sys}}^w \) and \( P_{\text{sys}}^b \) are received for indices \( k_1, k_2 \in I_i \), systemic default is caused by poor investments from banks. If \( k_1, k_2 \in I_g \), systemic default is related to unsuccessful governmental policy\(^{10} \). When \( k_1 \in I_i \cup I_s \) and \( k_2 \in I_g \cup I_s \) mixed strategies from sovereigns and institutions are required to minimize systemic default probability in the network.

Given the systemic default probabilities defined for different scenarios, the risk on the systemic default probabilities can be defined as

\[ \text{RISK} = P_{\text{sys}}^b - P_{\text{sys}}^w. \]

The risk measure, \( \text{RISK} \), defined in equation (9), defines the uncertainty of systemic default in the presence of multiple equilibria. The level of \( \text{RISK} \) depends on the network structure, bank returns, \( z_j^\pm \), and sovereigns, \( c_j^\pm \).

\(^{10}\) Acharya et al. (2014a) investigated how a government implementing a bailout triggers credit spreads and found a positive relationship between the level of government debt and credit risks.
Definition 6. A pair of equilibria \( d^* = \{d^{sys}(u), d^{sys}(u)\} \) is efficient if and only if \( RISK(d^*) \) is the lower bound across all possible equilibria \( d^{sys}(u) \).

\( RISK \) depends on network topology, meaning that is can be used to identify efficient equilibria (see Definition 6). In the optimistic scenario, when all entities can meet their liabilities and there are no defaults, there exists a trivial solution of system (3) and \( RISK = 0 \).

Alternatively, the efficient equilibrium state minimizes all creditors’ aggregate loss. Given a realization of shocks \( u_j \), the losses of creditors caused by a default of entity \( j \) are equal to total liabilities, \( l_j \). The expected loss suffered by the network as a result of entity \( j \) defaulting, \( EL_j \), is computed by aggregating the loss over the range of shocks in the case in which entity \( j \) defaults. Total expected systemic losses are

\[
EL^{sys} = \int \sum_j l_j d_j(u) p(u) du.
\]

Taking into account that several values of expected loss can exist in the case of multiple equilibria, we may analyze different scenarios. Hence, the Risk of Expected Loss (\( REL \)) can be defined as the difference between the highest and lowest expected losses:

\[
REL = EL^{sys} - EL^{sys}.
\]

Efficient equilibria are identified by minimizing \( REL \) for a given probability distribution of shocks, \( p(u) \).

The proposed framework makes evident how combinations of events in private investment and sovereign debt markets may place additional stress on banking networks. There may be less heterogeneity in sovereign debt market options available to the banks than in investments. That is, although the failure of a relatively small investment opportunity can cascade and cause financial stress in the Acemoglu et al (2015) model, there are in practice fewer sovereign bond investment opportunities available for banks. Thus, a haircut in the sovereign debt market is likely to cause a simultaneous common shock to a number of entities, providing a further means of amplifying a crisis via the network. Moreover,
the stochastic shock and its variance are important quantities that impact both sovereign and bank returns and may cause cascades of defaults in the network. This motivates the development of the novel econometric framework presented in Section 4.

3 Data and Summary Statistics

Modeling the interconnections between financial institutions is hampered by data availability. On the one hand, many of the theoretical frameworks are expressed in terms of inter-entity flows. However, these data are exceedingly difficult to obtain, particularly outside the official family; a good example is the UK interbank network in Giraitis et al (2016), who use data available to the Bank of England. On the other hand, there is a strand of literature that takes advantage of market-based data as proxies to develop an understanding of the interconnectedness of networks, as in, for example, Billio et al (2012) and Merton et al (2013). A recent work by van de Leur and Lucas (2016) finds that these interconnectedness networks based on market data produce valuable information that is not offered by alternative approaches. The work in this paper draws on the market-based data tradition in this literature.

Five-year CDSs are the most commonly issued and traded asset in this class and are the most liquid (Duca and Peltonen 2013, Pan & Singleton 2008, Kalbaska and Gatkowsi 2012); data on these contracts were extracted from Markit over the period from January 1, 2003, to November 21, 2013. Over the full period, there are 2842 end-of-day CDS spread prices for each sovereign and institution. The combined dataset contains 40 individual sovereigns and 67 institutions, for a total of 107 nodes used in the analysis, as listed in Tables 1 and 2.

The sample is divided into three separate phases; Phase 1 represents the non-crisis period from January 1, 2003, to September 14, 2008. This is typical of dating conventions used in literature to separate the pre-crisis and crisis periods; see the review of dates extant in the literature in Dungey et al (2015). Phase 2
represents the period from September 15, 2008, to March 31, 2010, consistent with the global financial crisis (GFC) and period following. The end of March 2010 represents the period prior to which the Greek debt crisis became critical in April 2010. Phase 3, from April 1, 2010, to November 21, 2013, represents the period of the Greek and European sovereign debt crises.

The first panel of Table 3 shows summary statistics for Phase 1. Phase 1 is the longest of the three exogenously chosen time periods, containing 1488 observations per entity. Latin America displays a higher mean spread during Phase 1, while financial institutions and insurance companies exhibit relatively higher kurtosis than other groups.

The GFC period, Phase 2, is the shortest of the sub-sample phases, with 403 observations per entity. There is an increase in spread means for most groups of institutions and sovereigns, reflecting the perceived increase in risk during this turbulent period in international debt markets. The financial institution group has the largest mean, standard deviation and kurtosis during Phase 2.

The third phase, associated with the Greek debt crisis and subsequent European debt crisis, involves a small decline in spread means; however the Eurozone group’s mean spreads increase from Phase 2, potentially due to the transformation of the Greek debt crisis into the European debt crises during the third phase. Insurance companies and Latin American sovereigns exhibit high levels of kurtosis compared with other groups.

CDS spreads were found to be non-stationary, I(1), with a maximum of one unit root according to KPSS and ADF tests. Moreover, the presence of heteroskedasticity in daily spreads was confirmed by applying Breusch-Pagan and White tests.

4 Econometric Framework and Hypotheses

4.1 Establishing network edges via Granger causality

Banks and sovereign debt issuers form network nodes linked by edges. The use of Granger causality tests on CDS spreads to establish edges between nodes has
a number of advantages in this framework. It is directly comparable to existing empirical networks of Billio et al (2012) and Merton et al (2013). It establishes directional edges, allowing for an examination of the causation from sovereign debt to banking markets. Granger causality established edges map clearly to the existing empirical frameworks for measuring and testing contagion during financial crises via the formation and breaking of linkages (the overarching framework for this is provided in Dungey et al., 2005).

To take into consideration the common stochastic trend(s) between the I(1) CDS series, a Vector Error Correction Model (VECM) is used:

\[
\Delta Y_t = \alpha \beta' Y_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t, \tag{12}
\]

where \( Y_t = [Y_{1,t}, ..., Y_{n,t}]' \), \( \Delta Y_{t-j} = Y_{t-j} - Y_{t-j-1} \) and \( \alpha, \beta, \Gamma \) are the parameters of the model\(^{11} \). The rank of the matrix \( \Pi = \alpha \beta' \) is estimated applying Johansen test and imposing the triangular restrictions of Phillips (1991). The parameters of model (12) are obtained by applying OLS.

CDS spread data used to motivate the Granger causality testing effectively represent a premium for insurance against the default of a third party. CDS spread prices reflect a perceived risk of default; favorable news decreases the value of the CDS spread, while unfavorable news increases the value. Significant Granger causality from entity \( i \) to entity \( s \) indicates that \( Y_i \) has at least one significant lag predicting the value of \( Y_s \). Thus, perceived risk of entity \( i \) defaulting predicts the perceived risk of default of entity \( s \). The edges of the network constructed from these Granger causality links represent predictors of each node’s perceived risk of default.

Once a VECM in (12) is estimated\(^{12} \), it can be represented as a VAR

\[
Y_t = \sum_{j=1}^{k} \Phi_j Y_{t-j} + \varepsilon_t, \tag{13}
\]

with cross-equation restrictions \( \Phi_1 = \alpha \beta' + \Gamma_1 + I_n \), and \( \Phi_j = \Gamma_j - \Gamma_{j-1} \), \( j = 2, 3, ..., p \). Granger causality between CDS spreads \( Y_i \) and \( Y_s \) can be assessed

---

\(^{11}\)A constant term is suppressed for simplicity.

\(^{12}\)The VECM is estimated using three lags, based on the AIC.
using the Wald test

$$WT = [e \cdot \text{vec}(\hat{\Pi})]'[e(\hat{\Pi} \otimes (Y'Y)^{-1})e]'^{-1}[e \cdot \text{vec}(\hat{\Pi})],$$

(14)
in which $Y$ is the matrix of independent variables from (13), vec$(\hat{\Pi})$ denotes the row vectorized coefficients of $\hat{\Pi} = [\Phi_1, ..., \Phi_k]$, $\hat{\Pi} = T^{-1} \sum_{t=1}^{T} \hat{\epsilon}_t \hat{\epsilon}_t'$ and $e$ is the $k \times 2(2k + 1)$ selection matrix

$$e = \begin{bmatrix}
0 & 1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & 0 & 1 & \ldots & 0 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & 1 & 0 & \ldots & 0 & 0 \\
\end{bmatrix}.$$

Each row of $e$ selects one of the coefficients to set to zero under the non-causal hypothesis $Y_i \rightarrow Y_s$.

The empirical contagion literature typically focuses on changes in the structure of short-term relationships across two periods. Consider, for example, the first-period interaction matrix estimated for a non-crisis period, denoted $\Phi_1^{nc}$, and a crisis period, denoted $\Phi_1^{cr}$, as follows:

$$\Phi_1^{nc} = \begin{bmatrix}
\phi_{ni,1}^{nc} & \phi_{ns,1}^{nc} \\
\phi_{si,1}^{nc} & \phi_{ss,1}^{nc}
\end{bmatrix},$$

$$\Phi_1^{cr} = \begin{bmatrix}
\phi_{ni,1}^{cr} & \phi_{ns,1}^{cr} \\
\phi_{si,1}^{cr} & \phi_{ss,1}^{cr}
\end{bmatrix}.$$ Tests for changes in the network finance literature (and related tests for contagion) can be characterized as tests of whether $\phi_{is,k}^{nc} = \phi_{is,k}^{cr}$ and $\phi_{si,k}^{nc} = \phi_{si,k}^{cr}$ for all $k$.

In this paper, the focus is on the formation of new links:

- new link from $Y_i$ to $Y_s$ $H_0 : \phi_{is,k}^{j-1} = 0; \phi_{si,k}^{j} \neq 0$ (15)
- new link from $Y_s$ to $Y_i$ $H_0 : \phi_{is,k}^{j-1} = 0; \phi_{is,k}^{j} \neq 0$ (16)

and the breaking of existing links

- broken link from $Y_i$ to $Y_s$ $H_0 : \phi_{si,k}^{j-1} \neq 0; \phi_{si,k}^{j} = 0$ (17)
where index \( j \) is assigned to a phase; non-crisis, GFC or European sovereign debt crisis.

The results of the Wald test indicating Granger causality are recorded as binary entries in matrix \( A \) as

\[
A = [a_{is}],
\]

where

\[
a_{is} = \begin{cases} 
0, & \text{if } Y_i \text{ does not Granger cause } Y_s \\
1, & \text{if } Y_i \text{ Granger causes } Y_s
\end{cases}
\]

Matrix \( A \) is used to construct the directional edges between sovereigns and banks.

### 4.2 Network connectedness

Once linkages between institutions and sovereigns, represented by matrix \( A \) are established, the strength of these linkages can be quantified by assigning weights \( W = [w_{ij}] \) to network edges\(^{13}\). Using the VAR from equation (13) as an approximating model, weights \( w_{ij} \) can be obtained from variance decompositions, as proposed by Diebold and Yilmaz (2009). Suppose that \( j \)'s contribution to entity \( i \)'s \( H \)-step-ahead generalized forecast error variance, \( \theta_{ij}^H(H) \), is

\[
\theta_{ij}^H(H) = \frac{V_{jj}^{-1} \sum_{h=0}^{H-1}(e_i'B_hV e_j)^2}{\sum_{h=0}^{H-1}(e_i'B_hV B_i'e_j)}, \quad H = 1, 2, 3, \ldots,
\]

in which \( V \) is the variance-covariance matrix for the error vector \( \varepsilon_t \), \( V_{jj} \) is the standard deviation of error term \( j \), and \( e_i \) is the selection vector with one as the \( i \)th element and zero otherwise. The coefficient matrices, \( B_i \), obey the recursion

\[
B_i = \Phi_1 B_{i-1} + \Phi_2 B_{i-2} + \ldots + \Phi_k B_{i-k}, \quad B_0 \text{ being an } n \times n \text{ identity matrix and } B_i = 0 \text{ for } i < 0.
\]

Note that the generalized variance decomposition allows for correlated shocks and does not depend on the ordering of the variables.

\(^{13}\)In this case, the network is defined as a weighted directed graph. A weighted financial network is also used by Demirer, Diebold, Liu and Yilmaz (2015) and Glasserman and Young (2015) to model connectedness between financial institutions.
In the original generalized framework of Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998), variance shares do not necessarily sum to 1, that is, \( \sum_{j=1}^{n} \theta_{ij}^g(H) \neq 1 \). Hence, each entry of the generalized variance decomposition matrix is normalized by the row sum as

\[
    w_{ij} = \frac{\theta_{ij}^g(H)}{\sum_{j=1}^{n} \theta_{ij}^g(H)}.
\]

Now, by construction, \( \sum_{j=1}^{n} w_{ij} = 1 \) and \( \sum_{i,j=1}^{n} w_{ij} = n \).

Given the estimates of matrix \( A \) and weighting matrix \( W \), the structure of the weighted network can be characterized by matrix

\[
    \tilde{A} = A \odot W,
\]

where \( \odot \) is the Hadamard product. Elements of adjacency matrix \( \tilde{A} \) capture the connectedness between institutions and sovereigns conditional on significant causal linkages between them\(^{15}\). The network defined by adjacency matrix \( \tilde{A} \) shows the predictors of the risk of default subject to a shock captured by matrix \( W \). Using the entries of matrix \( \tilde{A} \), system-wide completeness is measured as

\[
    C = \frac{\sum_{i,j=1, i \neq j}^{n} \tilde{a}_{ij}}{\sum_{i,j=1, i \neq j}^{n} w_{ij} \cdot \sum_{i,j=1}^{n} w_{ij}}.
\]

This measure is used in the following sections to analyze the system-wide connectedness between the financial institutions and sovereigns\(^{16}\). The proposed econometric framework permits us to formalize the following empirical hypotheses.

**Hypothesis 1** The strength of links between nodes changes during periods of stress. This hypothesis relates to the test of whether new links form or links are removed due to the forces of contagion described in equations (12) to (15).

\(^{14}\)Matrix \( W \) is not necessarily symmetric, in contrast to the partial correlation network of Anufriev and Panchenko (2015), which is symmetric by construction.

\(^{15}\)This approach extends the spillover index of Diebold and Yilmaz (2009). While the spillover index contains all elements of a variance decomposition matrix, here, the elements that are not linked causally are equal to zero. The importance of disentangling the network strength from the network structure is also highlighted by Scida (2015) in a different context.

\(^{16}\)The completeness measure can be computed for sub-networks (e.g., the completeness of a specific geographical region) in the same way by summing up the specific elements \( a_{ij} \) and \( w_{ij} \).
Hypothesis 2  The number of links changes during a period of stress. This test is compatible with the results in papers such as Billio et al (2012), who observe that networks increase in density during stressful periods.

Hypothesis 3  The completeness of the weighted network increases during stressful periods. This hypothesis will distinguish the results of the role of changes in both the number and strength of linkages to determine whether networks are in fact more intertwined during periods of stress.

Formally, we use results from the Granger causality tests in each sub-period between each of the nodes to assess whether the strength of links between nodes has changed. Given the estimates of matrices $\mathbf{A}$ and $\mathbf{\tilde{A}}$, hypotheses 1 and 2 can be tested formally by applying the statistical test of Mantel (1967). The null hypothesis that networks in two different phases are identical is tested using the following statistic

$$Z = \sum_{i,k=1}^{n} a_{ik} a_{ik}^{-1} \quad \text{for Hypothesis 1,}$$

$$Z = \sum_{i,k=1}^{n} a_{ik} a_{ik}^{-1} \quad \text{for Hypothesis 2,}$$

in which index $j > 1, 2, 3$ is assigned to designate the phases. The null distribution of $Z$ or $Z$ is obtained by a finite population approach outlined by Mantel (1967).

5 Results

To illustrate the degree of connectivity in the financial network, Figure 1 represents the network of significant Granger causality links between pairs of financial institutions in Phase 1. This network is extremely dense, making it difficult to derive any meaningful analysis of these results other than confirming the high degree of connectivity in these markets. (Note these networks include the US - to counter the possibility that a common market factor is driving our result
we conducted the same analysis using US spreads as a control variable in the Granger causality tests, with no discernible difference. Consequently, we analyze the specification including the US as a node to provide a comparable analysis for all geographic sources of connectivity.) The high level of connectedness is consistent with the discovery of a major common global factor in CDS spreads in Longstaff et al (2011) and Eichengreen et al (2012). Due to the difficulty of analyzing highly interconnected nodes, we do not present the combined network of financial institutions and sovereigns. In both cases, the degree of connectivity is relatively high - the potential number of links is 67!/65!(=4355) links in the financial sector network, and 107!/105!(=11342) in the combined network.

We first consider the results for the financial network then those for the combined financial and sovereign network. For brevity, we record first that in each case, Hypotheses 1 and 2 are accepted by the empirical tests at standard significance levels - there is no evidence the networks are unchanged between different phases of the sample.

5.1 Financial Institutions network

To aid analytical tractability, we condense the network shown in Figure 1 to five nodes. The 67 financial institutions in our sample are grouped into institutional types: banks, insurance companies, investment banks, real estate firms and other financial institutions. The constituents of these groups are shown in Table 1. The dispersion of these institutions by country is not conducive to undertaking a geographic-institutional breakdown; as we are considering institutions involved in the CDS market we make the relatively safe assumption these institutions are globally active investors. Institutions may invest in almost any sovereign debt market and be involved in cross-border counterparty arrangements and have sophisticated currency hedging mechanisms in place. A potential limitation of our approach is home bias or incomplete currency hedging distorting the results.

\footnote{An analysis of the changing connections between financial institutions in Europe and the US using equity market data may be found in Diebold and Yilmaz (2016) and for global banks in Demirer et al (2015). Moreover, a number of papers consider the detailed relationships for CDSs within these regions. For example, see Fabozzi et al (2016) for the Eurozone.}
Figure 2 presents the same information as Figure 1 using the institutional types as nodes, displaying the same high degree of completeness. The width and shade of the edges indicate the strength of links between two nodes, representing the proportion of significant linkages among potential linkages, as explained in Sections 4.1 and 4.2. Figure 2 illustrates the strength of the links involving banks, financial institutions and insurance companies, while links to real estate and investment firms are less strong. Arrows on the ends of edges provide evidence on direction of transmission - the results suggest bidirectional linkages.

Table 4 documents that 2795 of the 4355 potential links exist in Phase 1, decreasing to 2583 in Phase 2, confirming Hypothesis 2. Between Phase 1 and Phase 2, the net reduction in links is due to the loss of 1118 links, overwhelming the formation of 906 slightly stronger links (the average strength of the new links is 0.0142 compared with that of the removed links of 0.0127). Thus, between Phases 1 and 2, the completeness of the weighted network of financial institutions falls from 71.41% to 64.21%, primarily through a reduced number of links. (See also Eichengreen et al (2012), who find that spillovers from US to European banks decrease during the GFC period.) The further removal of 1718 (stronger) links and formation of 579 (weaker) links between Phase 2 and Phase 3 means that the net loss of 1139 links results in weighted network completeness falling to 36.15%, again primarily due to a decreased number of links. Overall the Phase 3 network has 1351 fewer links than Phase 1, and the average strength of links has increased. This important finding confirms the fragility of the financial sector during the GFC; see also Alter and Shuler (2012). Another important conclusion is that once both the existence and strength of linkages are taken into account, the completeness of the financial network is lower in Phase 2 compared with Phase 1, which differs from the findings of Diebold and Yilmaz (2016), and lower in Phase 3, which is consistent with Caporin et al (2014).

Figure 3 illustrates the location and strength of newly formed and removed linkages for each phase. The reduction in links is distributed relatively evenly across financial institutions in Phase 1 and Phase 2 (Figure 3 a), reflecting the
general market conditions, rather than a specific institutional type common to the global sample. The removal of links between banks and insurance companies is also indirectly observable through an investment channel by unravelling the strong direct connections from banks to investment institutions and from investments to insurance. This pattern is part of the complex debate surrounding whether insurers were causal in generating the systemic risk of this period; see Biggs and Richardson (2014).

Between Phase 2 and Phase 3, there is evidence of disconnection between investment firms and real estate companies. Banks and financial institutions are the focus of a substantial number of disconnections during the crisis, and this may reflect new international risk assessments and domestic regulatory environments whereby financial companies are recognized as contributing to systemic risk. However, the removed links from financial companies to insurers highlight the further propagation of systemic risks. Insurers favor self-regulation and insurance as a recipient of shocks from banks; see Cummins and Weiss (2014), which constrasts with the view of the Financial Stability Board, and Acharya et al (2014b), who report that insurers may propagate systemic risk.

As a proportion of total links, there are relatively few new links forming during Phase 2 (Figure 3c). As institutions attempt to manage their portfolios, and risk appetite generally decreases, the financial system becomes less interconnected. Regulators around the globe have more carefully monitored financial institutions since 2008, and formed new bodies to address segments of the financial sector, which may have been a contributing factor.

5.2 Combined financial institutions and sovereign debt network

The combined financial institutions and sovereign network has potentially $107!/105! (=11342) links. The number of links in Phase 1 is 5886, increasing to 8862 in Phase 2 and falling to 4184 in Phase 3; see Table 4. The intertwining of these two sectors is relatively incomplete in the pre-crisis period (at 62.61%) but rises substantially during the first crisis sample. In this manner, the Phase 1 network
is robust according to the definitions of Gai and Kapadia (2010) and Acemoglu et al (2015). The crisis periods also reflect the increases in linkages found in Billio et al (2012) and Merton et al (2013). Network link changes seem relatively large but must be seen in the context of total number of links; in total 2567 (22.85% of all possible links) were unchanged in the unweighted network during the sample period, 1647 (14.66%) links remained present and intact, and 920 (8.19%) links did not exist at any point in the sample. However, between Phase 1 and Phase 2, the network lost 1422 and gained 4198 links, a net gain of 2776. The average strength of the formed links in the weighted network was 0.0088, weaker than that of the lost links of 0.0092. That is, the increased completeness between Phases 1 and 2 is due to the formation of more weaker links in the presence of declining strong links. In the transition from Phase 2 to Phase 3, a further 5479 links, of average strength 0.0091, were lost and 1001 formed, of average strength 0.0102. The net loss of 4478 links were of lower average strength than those gained, such that overall the number of links fell, contributing to the decline in weighted network completeness.

Not only do the proportions of links change between phases, supporting Hypothesis 2, but the taxonomy of these changes is highly revealing. Categorizing nodes into geographic sovereign debt markets and financial institution types, as in the previous sub-section, Figure 4 provides the schematic for links that are broken and formed from Phases 1 to 2, and from Phases 2 to 3.

The results in Figure 4(a) show that the CDS premia for insurance companies became disconnected from North American sovereign debt CDS premia during the first crisis phase. There is a concentration of lost connections between the North American and Euro sovereign nodes and with investment sector links. The Eurozone’s disconnection from Europe and Asia is less evident in the link changes. However, there is some evidence of disconnection between bank CDS premia and sovereign debt, perhaps speaking to the intimate connection that US sovereign debt has with bank balance sheets, the US Federal Reserve’s liquidity provisions, and feedback effects between sovereigns and banks posited in Acharya et al (2014a).
Relatively more links are formed between Phase 1 and Phase 2, as shown in Figure 4(c). Most pronounced are the new links between North American, African, Asian and Latin American sovereign bond markets, suggesting an increasing importance of developing markets in global risk determination during this phase. There are also considerable increases in investment firm linkages with European and North American sovereign markets.

During Phase 3, the links previously established between North American, African, Asian and Latin American sovereigns are largely undone; see Figure 4(b). There is less evidence of a retraction of links with financial sector nodes, although the reduction of linkages with financial firms is relatively strong. Newly formed links in this phase, as shown in Figure 4(d), relate to North America, reflecting new influences on investments and Eurozone from North American entities; this may reflect increasing market involvement by non-European institutions, including Goldman Sachs and Metlife. Further influences on new connections may include asset write-downs, increased premia, regulator scrutiny of sovereign debt exposure and feedback effects between sovereign debt markets and banks (and, by extension, the financial sector).

The combined network is characterized by highest completeness during crisis Phase 2, originating from financial institutions. Moreover, the net number of new links is positive for Phase 2 and negative for Phase 3, consistent with results reported by Diebold and Yilmaz (2014) for the global equity market; they show that system-wide connectedness is significantly higher during the GFC. The difference between our work and existing network papers based on Granger causality is the use of weighted networks, revealing that although the number of unweighted links may increase during a crisis, when the links are weighted by their relative strength, the completeness of the network may not decline by as much as in an unweighted system (and in some cases, the weights may be sufficient to induce an increase in network completeness).18

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18Recent work by Pesaran et al (2016) shows how a network can be characterized by $\delta_i$, the estimated degree of pervasiveness, based on the weighted column sums of the adjacency matrix (normalized by row). The most dominant node in the network has the largest value of $\delta_i$, subject to the caveat that $\delta_i \geq 0.5$ for the existence of a valid network effect. The
5.3 Identifying the networks in different scenarios

In the previous discussion, the sensitivity of sovereigns and banks to defaults was found to be a key characteristic for identifying the structure of the network. Following Definition 5, the sensitivity of entities to default defines the expected number of defaults in the network, which in turn can be used to uncover the network structure.

Clearly, the sensitivity of a counterparty to default indicator $scd_{j,i}$ from equation (4) is closely related to an impulse response function. If the direction of shock $u_j$ is positively correlated with the number of defaults (i.e., a large negative shock increases the probability of an entity to be insolvent), measured by vector $u_j$, impulse responses can be used as empirical measures of $scd_{j,i}$. In this case, the expected number of defaults can be estimated from significant Granger causality linkages and the respective variance decompositions, as discussed in Section 4.

**Definition 7** A network $G^1$ is more fragile than $G^2$ if for each entity $i$, the expected number of defaults introduced in Definition 5 is higher under $G^1$ than under $G^2$.

As follows from Definition 719, the fragility of the network can be assessed based on the expected number of defaults, which is in turn defined by significant Granger causality linkages and the respective variance decompositions. Given an external shock $u_j$, the expected number of defaults can be estimated under the different scenarios related to different values of $q_j$ (see Section 2).

To investigate the fragility of the network in the three phases, probabilities of the different scenarios posited in our theoretical framework (Good times, Stress, Poor investment, Poor government) are estimated. We use estimates from equation (13) for each phase to generate recursively one-step ahead forecasts. The application of this framework to our network provides evidence of weakly dominant banking and insurance sectors in the first phase ($\delta_{bank,1} = 0.67, \delta_{insurance,1} = 0.50$) and weakly dominant banking in the following two phases ($\delta_{bank,2} = 0.56, \delta_{bank,3} = 0.74$). None of the sovereign nodes become dominant in any phase, although the value of $\delta_i$ for European and Euro located sovereigns approaches 0.5 in Phase 3. Consequently, the results of this approach are aligned with the results presented in the paper.

19This definition is similar to Zhou (2016).
predicted values of the CDS spreads are used to estimate the probabilities of different scenarios. In particular, in each phase, probabilities $P r^+ = m^+/m$ and $P r^- = m^-/m$, where $m^+/-$ is a number of predicted values for a higher/lower than an average predicted CDS spread, can be interpreted as probabilities of high and low default risks, respectively. These probabilities can be used in each phase to obtain an aggregated shock conditional on a specific scenario:

- **Good times:** $u_j = P r^+(S) \cdot u_j(S) + P r^+(F) \cdot u_j(F)$,
- **Stress:** $u_j = P r^-(S) \cdot u_j(S) + P r^-(F) \cdot u_j(F)$,
- **Poor investment:** $u_j = P r^+(S) \cdot u_j(S) + P r^-(F) \cdot u_j(F)$,
- **Poor government:** $u_j = P r^-(S) \cdot u_j(S) + P r^+(F) \cdot u_j(F)$.

where $S$ is assigned to sovereigns and $F$ to financial institutions. These aggregated shocks are used to obtain weights, $w_{ij}$, of variance decompositions that allow us to identify the structure of the combined network.

The expected number of defaults for the combined network conditional on the size of the aggregated shock, $u$, are presented in Figure 5 for each of the scenarios. Figures 5(a) to (c) show the impact of scenarios based on **Good times**, **Stress**, **Poor investment**, and **Poor government** shocks of increasing magnitude (up to 3 standard deviations) across the horizontal axis. Although stress scenarios always produce the largest number of expected defaults and good times the lowest, the degree of expected defaults varies considerably across the phases. In Phase 1, good times result in a probability of fewer than 5 defaults, rising to almost 50 in Phase 2 and returning to approximately 5 in Phase 3. However stress conditions see expected defaults go from over 25 in Phase 1 to more than 100 in Phase 2, before reducing to a smaller number of fewer than 10 in Phase 3. The third phase represents a rebound to a lower default probability than Phase 1 in stress conditions – perhaps representing the loss of already non-resilient entities.

A further feature is that during Phase 1, shocks due to poor investment outcomes are more likely to result in default than those due to poor government.
Both of these increase dramatically to approximately 80 defaults in Phase 2 in either case. By Phase 3, we have a slightly higher probability of default outcomes anticipated for the entities in the face of poor government shocks than for poor investment shocks. Whether this is a pattern, emerging in the aftermath of other major crises because of post-crisis risk reassessment, is a topic worthy of further investigation. It implies that recent experience with private investment sector shocks may reduce the likelihood of future large private investment shocks generating large numbers of defaults. These results shed light on debate in the literature over whether bank or sovereign default precedes the other in an orderly manner - the results of this work suggest that it is the intertwining of these markets that provides protection against financial fragility. This supports the vital importance of adopting a combined approach to systemic risk as, particularly during periods of stress.

The empirical work in this paper provides strong evidence of the changing strength of linkages between nodes in the network of financial institutions and sovereigns during periods of stress, consistent with the existence of contagion, as argued in Hypothesis 1. The number of links between nodes changes from between Phases 1 and 2, increasing the density of the unweighted combined network, which is consistent with findings in the existing literature and supports Hypothesis 2. The completeness of the weighted financial network decreases between Phases 1 and 2 but declines dramatically in Phase 3. Our evidence suggests that in these markets completeness was reduced for all networks between Phase 1 and 3. The changing completeness of the combined network in the different phases and under different scenarios represents changes in the structure and combination of what Acemoglu et al (2015) classify as a $\gamma$-convex combination of networks. Overall, less diversified patterns of linkages are more fragile during crisis times but could be more robust when normal times prevail.
6 Conclusion

This paper investigates international connections between financial institutions and sovereign debt markets with both a theoretical framework and an empirically tractable implementation. We extend the model of Acemoglu et al (2015), which shows how financial institutions facing shocks generated from real economy investments may demonstrate a robust-but-fragile network subject to increased risk of default when faced with either a sufficiently large shock (or contemporaneous small shocks). Our innovation is to extend uncertainty affecting returns to sovereign bond markets as the alternative investment option to the risk modelled for the real economy investments. By including potential haircuts on sovereign debt investment, we address the debate on the importance of links between financial institutions and sovereign debt under crisis conditions.

The results reinforce the ‘robust-but-fragile’ nature of networks of financial institutions facing real economy shocks and emphasize that sovereign debt market haircuts may additionally cause simultaneous shocks to a number of institutions, providing a means of amplifying uncertainty via the network. This uncertainty implies different scenarios for network changes: Good times, in which both returns in the real economy are good and there are no sovereign defaults in which case the network is relatively robust; Poor investment, in which returns in the real economy are poor but there are no sovereign defaults, consistent with potential stress under the ‘robust-but-fragile’ analysis of Acemoglu et al (2015); Poor government, in which returns in the real economy are good but poor government policy leads to sovereign bond haircuts, these may lead to financial system fragility; and Stress conditions, in which both poor returns in the real economy and sovereign haircuts place stress on the financial network.

These scenarios permit identification of the sources of changes to the network’s systemic default probability and estimate the expected number of defaults in the combined network. Our findings speak strongly in favor of the necessity of mixed strategies by sovereigns and financial institutions to minimize the expected number of defaults in the network during the GFC and European crisis.
The paper specifically examines the transition of a combined network of sovereigns and financial institution and one of financial institutions during the period 2003-2014 through three phases. In the transition between Phase 1 (representing non-crisis conditions) and Phase 2 (which corresponds to the GFC), the network shows evidence of changing linkages between nodes - both the removal of existing links and the formation of new links, consistent with the contagion literature. Between Phases 1 and 2, the net number of links in the combined network increases; the sheer growth in the number of linkages is enough to increase weighted network completeness, consistent with the greater density of links observed in much of the existing work on financial networks during periods of crisis. Between Phases 2 and 3, corresponding to the European sovereign debt crisis, the number of links declines significantly, demonstrating recovery from the GFC. The disentangling of the strength and existence of linkages between nodes reveals that the completeness of the weighted network may decrease, consistent with the existence of contagion and increasing numbers of links. It provides confirmatory evidence of the robust-but-fragile nature of the network during periods of crisis.

Examining changes in the networks by geographical region and type of institution reveals that during the transition from the pre-crisis Phase 1 to Phase 2 of the GFC, there is a significant formation of links, which mainly centers on the Eurozone, North and Latin America and investment companies. This follows expectations, as the GFC was closely linked to the collapse of the United States subprime mortgage market. In the transition from Phase 2 to Phase 3, the Greek/European debt crisis, most link changes are centered on North America and the Eurozone. A large number of links are formed from North America to investment companies and to the Eurozone.

The next step in this agenda is to test for evidence of key links between sovereigns and financial institutions in terms of δ-dependency, Acemoglu et al (2013), which identifies whether there are critical sets of links that influence the financial fragility of the system.
7 Appendix

7.1 Proofs

Proof of Proposition 1.

This trivially follows from the definition of \( \tilde{q}_j : (\mathbb{R}, +\infty) \to [-1, 1] \).

Proof of Proposition 2. **Condition (a)**

Suppose that the network \( G \) does not contain any cycles. In this case the set of all nodes is

\[
N = \{ N_l \cup N_b \},
\]

\[
\varnothing = \{ N_l \cap N_b \},
\]

where \( N_b \) contains entities that only borrow money from their counterparties, and \( N_l \) is a set of lenders. If entities from \( N_b \) do not lend to anyone, for any realization of shock \( u \), there exists a unique threshold value, \( \tilde{q}_j^* \), for all \( j \in N_b \). Therefore, the set \( N_l \) contains all creditors of entities from set \( N_b \). The default conditions for all \( i \in N_l \) are determined by the borrowers and for this reason are also unique. This implies that the existence of a continuum of equilibria is only possible when there is at least one entity that simultaneously lends and borrows money, which contradicts (23). Hence, multiple equilibria can exist only in the cyclical network.

**Condition (b)**

Assume that there exists at least one cycle \( C_k \) in the network \( G \). It is sufficient to show that for a specific realization of shock \( u \), the network \( G \) has at least two equilibria. Consider the sensitivity to default measure, \( scd_{k,k-1} \), defined in (4). If \( scd_{k,k-1} \neq 0 \), then there exists a non-empty interval

\[
[a_i; b_i] = [\min(\tilde{q}_k(d_{k-1} = 0), \tilde{q}_k(d_{k-1} = 1)); \max(\tilde{q}_k(d_{k-1} = 0), \tilde{q}_k(d_{k-1} = 1))],
\]

which implies different values for the default conditions.

Consider two vectors of payments \((\hat{x}_1, ..., \hat{x}_k)\) and \((\bar{x}_1, ..., \bar{x}_k)\), that are asso-
cated with the threshold values

\[ q_1(x_1 = \hat{x}_1) = a_1 \]
\[ \ldots = \]
\[ q_k(x_k = \hat{x}_k) = a_k \]

and

\[ q_1(x_1 = \bar{x}_1) = b_1 \]
\[ \ldots = \]
\[ q_k(x_k = \bar{x}_k) = b_k \]

In this case each of \( k \) entities are linked by mutual liabilities. However, in this case shock \( u_i \) takes values on the interval \([a_i; b_i]\), which implies two equilibria, \( \hat{d} \) and \( \bar{d} \).

**References**


Table 1: Financial institutions grouped by broad type.

<table>
<thead>
<tr>
<th>Banks</th>
<th>Financials</th>
<th>Insurance</th>
</tr>
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<tbody>
<tr>
<td>Aust &amp; New Zld Bkg</td>
<td>ACOM CO LTD</td>
<td>ACE Ltd</td>
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<tr>
<td>Ameren Express Co</td>
<td>John Deere Cap Corp</td>
<td>Aegon N.V.</td>
</tr>
<tr>
<td>Barclays Bk plc</td>
<td>MBIA Inc.</td>
<td>American Intl Gp Inc</td>
</tr>
<tr>
<td>BNP Paribas</td>
<td>Natl Rural Utils Coop</td>
<td>Allstate Corp</td>
</tr>
<tr>
<td>Cap One Finl Corp</td>
<td>Aiful Corp</td>
<td>Aon Corp</td>
</tr>
<tr>
<td>Citigroup Inc</td>
<td>ORIX Corp</td>
<td>Assicurazioni Generali</td>
</tr>
<tr>
<td>Ctrywde Home Lns</td>
<td>Gen Elec Cap Corp</td>
<td>CHUBB CORP</td>
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Table 2: Sovereigns grouped by region. Groups are intentionally broad to minimize the total number.

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Table 3: Summary statistics are reported for all sovereign and financial institution CDS spread data used in this paper.

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<th>Phase</th>
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Table 4: This table contains statistics used in the analysis of network structures.
The average link strength is estimated from the connectedness of each respective network.
The number of edges was calculated using bivariate Granger causality tests between network nodes (entities). Completeness is calculated via equation (19).

<table>
<thead>
<tr>
<th></th>
<th>Formed</th>
<th>Removed</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Phase 1</td>
<td>Phase 2</td>
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<td>Completeness</td>
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<tr>
<td>Completeness</td>
<td>0.6261</td>
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Figure 1: This figure displays the network of financials in Phase 1 (01/01/2003 - 14/09/2008). Edges were calculated with bivariate Granger causality tests between financial institutions (nodes) at the 5% level of significance.
Figure 2: This figure shows a condensed version of the Phase 1 financial network from Figure 1. The changes are performed by grouping financial institutions/nodes into industries.
Figure 3: This group of figures displays changes in the financial network between Phase 1 (01/01/2003 - 14/09/2008), Phase 2 (15/09/2008 - 31/03/2010) and Phase 3 (01/04/2010 - 21/10/2013). Changes are calculated using matrix $A$. 

(a): Removed links Phase 1 to 2  
(b): Removed links Phase 2 to 3  
(c): Formed links Phase 1 to 2  
(d): Formed links Phase 2 to 3
Figure 4: This group of figures displays the combined sovereign and financial network changes. Changes between Phase 1 (01/01/2003 - 14/09/2008), Phase 2 (15/09/2008 - 31/03/2010) and Phase 3 (01/04/2010 - 21/10/2013) are calculated from matrix $A$. 

(a): Removed links Phase 1 to 2
(b): Removed links Phase 2 to 3
(c): Formed links Phase 1 to 2
(d): Formed links Phase 2 to 3
Figure 5: This set of figures shows expected number of defaults in the combined network for a shock size by multiples of standard deviations under the different scenarios. The logarithm of the CDS spreads is used for calculation.

(a): Phase 1 (01/01/2003 - 14/09/2008)

(b): Phase 2 (15/09/2008 - 31/03/2010)

(c): Phase 3 (01/04/2010 - 21/10/2013)