A Regular Demand System with Commodity-Specific Demographic Effects

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WITH COMMODITY-SPECIFIC
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by

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Abstract

Regular consumer demand systems almost invariably employ specifications that involve common functional forms in all equations. When applications involve cross-sectional data it is often the case that demographic effects are important. However it is plausible that demographic effects are commodity-specific. In this case, there may be a loss of efficiency if a common functional form across commodities is imposed artificially by entering redundant explanators in demand equations for which specific demographic influences are unwarranted. This paper explores an approach to specifying a complete system of demand equations which is fully regular but which nevertheless allows for commodity-specific variation in the functional form of the demand equations.

JEL Classification:
D11 - Consumer Economics: Theory,
D12 - Consumer Economics: Empirical Analysis
J1 - Demographic Economics

Key Words: Demographics, Demand System
I Introduction

This paper introduces and explores the implications of a system of consumer demand equations that allow for commodity-specific demographic effects. As is now standard, the approach utilises duality theory to derive a mutually consistent set of demand equations satisfying adding-up and, in principle, satisfying all other regularity conditions implied by consumer optimisation. Theoretically derived regular consumer demand systems almost invariably employ specifications that involve common functional forms in all equations. However it is plausible that demographic effects are commodity-specific. This raises the issue of whether commodity specific functional forms can be utilised while retaining integrability. In one sense the answer is obviously in the affirmative. If one is prepared to entertain sufficiently complex and non-linear functional forms, one could always specify an overarching general functional form which contains what appear to be special cases for specific commodity demands simply by imposing suitable exclusion restrictions on component variable parameter sub-functions. The approach we take in this paper may be interpreted in this manner, so in one sense it is not a radical departure from the norm. On the other hand, there can be a substantial loss of efficiency in econometric estimation if a common functional form across commodities is imposed artificially by entering redundant explanators in demand equations for which specific demographic influences are unwarranted. Furthermore, the interpretation of an artificial overarching function can be problematic. Our approach involves an effective compromise that retains ease of interpretation, allows reasonably complex forms for sub-functions that enter particular commodity specifications, and at the same time retains integrability of the full system.
Most of the theoretical and applied work in demographic demand systems has been in the pursuit of constructing equivalence scales for making welfare comparisons across households of varying demographic structure. Barten (1964) was the first to theoretically model demographics in demand by scaling each price by a commodity specific equivalence scale. His model has been criticised for its “excessive substitution effects” as demonstrated by Muellbauer (1977). Gorman (1976) added fixed costs that vary with household demographics to the household cost function of Barten that reduced much of this effect. These demographic fixed costs are the same as Pollak and Wales’ (1981) demographic translating and add intercept shifts in demands for each good that are dependent on demographic variables, as does Muellbauer’s (1976) demographic modification of his PIGL and PIGLOG models.

By scaling a reference household’s cost function by an equivalence scale that depends on demographics and their interaction with prices and utility, Ray (1983) demonstrated that demographics could alter both the level and shape of the expansion path for commodities. This is the approach adopted by Ray (1986) and Lancaster and Ray (1998) whose demographic generalisation of the Almost Ideal Demand (AID) System allows the level and slope (against log expenditure) of demand shares to be dependent upon demographics. Although dependence of the equivalence scale on utility violates the independence of base (IB) property - see eg Roberts (1980) - and rules out using the equivalence scale for valid welfare comparisons - see Pollak and Wales (1979) – the current paper is concerned with specifying a demographic demand system that can be used to obtain more accurate estimates of demand, rather than the identification of an equivalence scale. Hence IB is not imposed.
While non-linear demographic demand systems have been estimated, such as the Quadratic Almost Ideal Demand System (QUAIDS), the majority, such as Blacklow and Ray (2000) and Michelini (2001), have only allowed the demographic generalisation to affect the level of demand shares, in order to allow welfare comparisons with the resulting IB equivalence scales. Dickens, et. al. (1993), Blundell, et. al. (1993) and Pashardes (1995) allow for demographic variables to affect the intercept, slope and curvature of the demand shares, by allowing the parameters of the budget shares to vary across households. They find that the non-linear demand systems outperform their linear counterparts and can better identify equivalence scales.

The vast majority of demographic demand systems, including those referred to above, have limited demographic variables to the size and composition of the household, namely the number of adults and children and their ages. In addition the demographic variables have entered the demand equation for every commodity, whether appropriate or not.

Our demand specification is based on a modification of the Deaton and Muellbauer (1980) AID system. The modified system (MAIDS) is a variation on the specification of Cooper and McLaren (1992). The variant employed here nests the AID system when a certain parameter (the ‘MAIDS parameter’) is unity. Like the AID system, MAIDS is non-homothetic. MAIDS has attractive regularity properties, which, in combination with its non-homothetic characteristics, serve as a useful platform for examining demographic influences on demand. In particular, it is possible to allow for different demographic influences on the shape of expansion paths for various
commodities whilst retaining adding up across commodities. This is achieved by generalising the indirect utility function underlying MAIDS through introduction of demographic variables as explanators of price elasticities.

Section 2 sets out the underlying MAIDS model structure. Section 3 outlines the theoretical structure containing the demographic extension (DEMAIDS). Section 4 describes the data and details the specified commodity-specific demographic influences that we explore. Our estimation approach is briefly described in Section 5, followed by a discussion of the main results.

II The Underlying MAIDS Model

To begin, consider the following variant of the MAIDS indirect utility function (IUF):

\[
U(c, p) = \frac{c^{1-\eta}}{B(p)A(p)^{-\eta}} \ln \left( \frac{c}{P} \right)
\]  

where \( P \) is a translog price index given by

\[
\ln P \equiv \ln P(p) = \ln A(p) + \frac{1}{2} \sum_{k=1}^{n} \sum_{l=1}^{n} \gamma_{kl} \ln p_k \ln p_l,
\]

with \( \sum_{k=1}^{n} \gamma_{kl} = 0, \gamma_{kl} = \gamma_{lk} \), and where \( A(p) \) and \( B(p) \) are Cobb-Douglas price indexes which are homogeneous of degree one (HD1) in a vector of prices \( p \), i.e.:

\[
\ln A(p) = \sum_{k=1}^{n} \alpha_k \ln p_k, \quad \sum_{k=1}^{n} \alpha_k = 1 \quad \text{and} \quad \ln B(p) = \sum_{k=1}^{n} \beta_k \ln p_k, \quad \sum_{k=1}^{n} \beta_k = 1.
\]

This contains the AID specification as a special case when the MAIDS parameter \( \eta = 1 \). At this point (1) reduces to \( U(c, p) = (1/B^*(p))\ln(c/P(p)) \) where \( B^* = B/A \).
which is the AID system IUF. However, regularity is sacrificed with the AID system since $B^*(p)$ is not monotonic in $p$.

Application of Roy’s Identity generates share equations of the fractional form:

$$
S_i = \frac{\alpha_i + \sum_{k=1}^{n} \gamma_k \ln p_k + (\beta_i - \eta \alpha_i) \ln (c/P)}{1 + (1-\eta) \ln (c/P)}.
$$

(4)

The share equations imply $S_i = \alpha_i$ for a reference level of real income, defined to be such that $c = P$ provided that at the same time we normalise prices such that $p_i = 1$ at this point. This of course also implies the normalisation $c = 1$ at the reference income level. As real income increases the shares asymptote to:

$$
\lim S_i = \frac{\beta_i - \eta \alpha_i}{1-\eta}.
$$

(5)

For $\eta = 1$ (the AID system case) limiting shares do not exist. Regularity is best pursued by restricting $\eta < 1$. In this case positive limiting shares for necessities require that $\beta_i$ be ‘not too small’ relative to $\alpha_i$. This is simply an illustration of the fact that, apart from regularity in prices, ‘proportionality’ of the specification is also of relevance. For the static model of main concern here, this also covers the condition of non-negative (direct) marginal utilities. For practical purposes, this requires that the IUF be increasing in $c$. Now from (1):

$$
\frac{\partial U(c,p)}{\partial c} = \frac{c^{-\eta}}{B(p)A(p)^{-\eta}} \left[1 + (1-\eta) \ln \left(\frac{c}{P}\right)\right].
$$

(6)

Given $\ln \left(\frac{c}{P(p)}\right) \geq 0$, a sufficient condition for positivity of (6) (for finite $c$) is the restriction $\eta \leq 1$. While there are no second order restrictions on properties of the IUF in $c$ for static models, it is useful to obtain restrictions appropriate to allowing
the static MAIDS IUF to be embedded as an instantaneous utility function within a
time separable intertemporal model. Such ‘intertemporal properness’ is obtained by
restricting the range of $\eta$ to ensure concavity of the IUF in $c$. From (6):

$$\frac{\partial^2 U(c, p)}{\partial c^2} = \frac{c^{\eta-1}}{B(p)A(p)} \left[1 - 2\eta - \eta(1 - \eta)\ln(c / P)\right].$$  \tag{7}$$

Concavity requires that (7) be non-positive, a condition that is clearly satisfied if
$\eta = 1$. For the case $\eta < 1$, in general this requires a data-dependent restriction:

$$\ln(c / P) \geq \frac{1 - 2\eta}{\eta(1 - \eta)}. \tag{8}$$

Whenever real income is above the reference level, we have $\ln(c / (P(p))) \geq 0$ and a
sufficient condition for concavity in the case $\eta < 1$ is then that $\frac{1}{2} \leq \eta < 1$. For very
low levels of real income satisfaction of (8) may require $\eta$ to lie towards the upper
end of this range. In models requiring intertemporal embeddedness, the reference
level of real income needs to be set sufficiently low so as to ensure this. In the current
work, we analyse a static system so we do not pursue this further and simply choose
the median level of total expenditure as the reference level. That is, prior to
estimation, total expenditure is normalised on the total expenditure of the median
household. The commodity prices faced by the median household are then used to
normalise commodity prices.

III DEMAIDS: The Demographically Extended MAIDS Model

To provide a reasonably flexible demographic extension to MAIDS we retain the IUF
(1) and generalise the translog price index (2) to the form:
\[ \ln P = \ln P'(p,c,x) = \ln A(p) + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=0}^{k-1} \gamma_{ij} \ln p_k \ln p_t + \sum_{k=1}^{n} \sum_{j=1}^{d_i} \theta_{ij} x_{ij} \ln(p_k / c), \]  

(9)

where \( x_{ij} \) denotes the \( j^{th} \) \( k \)-specific demographic effect. The demographic effects in (9) enter in a form which is homogeneous of degree zero (HD0) in money and prices. It may be noted that the overall demography-specific price index \( \ln P \), now given by (9) rather than (2), is not HD1 in prices. This reflects the demography-dependence of price effects. The specification is regular in the sense that the IUF (1) retains the property of being HD0 in money and prices.

Given (9), application of Roy’s Identity to (1) now yields:

\[ S_i = \frac{\alpha_i + \sum_{k=1}^{n} \gamma_{ik} \ln p_k + \sum_{j=1}^{d_i} \theta_{ij} x_{ij} + (\beta_i - \eta \alpha_i) \ln (c / P)}{1 + \sum_{k=1}^{n} \sum_{j=1}^{d_i} \theta_{ij} x_{ij} + (1 - \eta) \ln (c / P)}. \]

(10)

In DEMAIDS the share equations at the reference level of real income and prices are:

\[ S_i = \frac{\alpha_i + \sum_{j=1}^{d_i} \theta_{ij} x_{ij}}{1 + \sum_{k=1}^{n} \sum_{j=1}^{d_i} \theta_{ij} x_{ij}}, \]

(11)

indicating the dependence of share \( i \) on each of \( d_i \) relevant (commodity \( i \) specific) consumer demographic characteristics \( (x_{ij}, j = 1,...,d_i) \), relative to all potential commodity-specific demographic influences on shares. Demographic influences on consumer spending patterns may be expected to be greater for lower income levels and to gradually become irrelevant as consumer real income increases. The DEMAIDS structure takes this into account. The share system (10) may be interpreted as a weighted average of (11) and (5), with weights given by
1 + \sum_{k=1}^{n} \sum_{j=1}^{d_k} \theta_{kj} x_{kj} \) and \((1 - \eta) \ln \left( \frac{c}{P} \right) \) respectively. To make this interpretation explicit we rewrite (10) using (5) and (11) to give:

\[ S_i = \left[1 - W \right] \left\{ \begin{array}{c}
\alpha_i + \sum_{j=1}^{d_i} \theta_{ij} x_{ij} \\
1 + \sum_{k=1}^{n} \sum_{j=1}^{d_k} \theta_{kj} x_{kj}
\end{array} \right\} + \frac{\sum_{k=1}^{n} \gamma_{ik} \ln p_k}{1 + \sum_{k=1}^{n} \sum_{j=1}^{d_k} \theta_{kj} x_{kj}} + W \frac{\beta_i - \eta \alpha_i}{1 - \eta} \] (12)

where the weights \( W \) are defined by:

\[ W = \frac{(1 - \eta) \ln \left( \frac{c}{P} \right)}{1 + \sum_{k=1}^{n} \sum_{j=1}^{d_k} \theta_{kj} x_{kj} + (1 - \eta) \ln \left( \frac{c}{P} \right)} \] (13)

and have the property that \( W = 0 \) at the reference level and \( W \to 1 \) as \( c/P \to \infty \).

System (12) demonstrates that DEMAIDS expenditure shares are a weighted average of demographic/price and asymptotic factors. As income rises above the reference level, less weight is given to the reference-level demographic effects and to current prices and more weight is transferred to the limiting shares - limits that are in fact the same as for MAIDS. Note the effect of the MAIDS parameter \( \eta \) in determining the extent to which demographic influences are filtered out as real income rises.

**IV Data**

In this paper we estimate DEMAIDS using Australian Household Expenditure Survey data pooled over five complete surveys from 1976 to 1999 and combined with ABS CPI commodity price data, giving 32541 observations in all. A full description of the data is available in Blacklow (2003). Each survey, with the exception of 1989, is augmented with state commodity price data, comprising nine commodity price
indexes, one for each commodity in the nine commodity system estimated, ensuring some price variation during survey years. Price data differentiated by state were not available for 1989, but price variation relative to other survey years is achieved through the intertemporal pooling of the expenditure surveys. Descriptive statistics are given in Table 1. This comprises the nine commodity shares; the logarithm of household normalised total expenditure; the logarithms of the nine normalised commodity prices; and five demographic variables on household size and structure.

**Table 1: Descriptive Statistics**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share 1: Food and Non-Alcoholic Beverages</td>
<td>0.213</td>
<td>0.099</td>
<td>0.000</td>
<td>0.918</td>
</tr>
<tr>
<td>Share 2: Accommodation</td>
<td>0.263</td>
<td>0.151</td>
<td>0.000</td>
<td>0.974</td>
</tr>
<tr>
<td>Share 3: Power</td>
<td>0.035</td>
<td>0.030</td>
<td>0.000</td>
<td>0.704</td>
</tr>
<tr>
<td>Share 4: Clothing and Footwear</td>
<td>0.053</td>
<td>0.071</td>
<td>0.000</td>
<td>0.842</td>
</tr>
<tr>
<td>Share 5: Transport</td>
<td>0.147</td>
<td>0.133</td>
<td>0.000</td>
<td>0.951</td>
</tr>
<tr>
<td>Share 6: Medical and Personal Care</td>
<td>0.061</td>
<td>0.058</td>
<td>0.000</td>
<td>0.853</td>
</tr>
<tr>
<td>Share 7: Entertainment</td>
<td>0.109</td>
<td>0.103</td>
<td>0.000</td>
<td>0.913</td>
</tr>
<tr>
<td>Share 8: Alcohol and Tobacco</td>
<td>0.050</td>
<td>0.064</td>
<td>0.000</td>
<td>0.694</td>
</tr>
<tr>
<td>Share 9: Miscellaneous</td>
<td>0.068</td>
<td>0.082</td>
<td>0.000</td>
<td>0.893</td>
</tr>
<tr>
<td>Log Total Expenditure</td>
<td>2.906</td>
<td>3.708</td>
<td>-4.399</td>
<td>8.973</td>
</tr>
<tr>
<td>Log Price 1</td>
<td>-0.271</td>
<td>0.573</td>
<td>-1.439</td>
<td>0.330</td>
</tr>
<tr>
<td>Log Price 2</td>
<td>-0.259</td>
<td>0.438</td>
<td>-1.221</td>
<td>0.205</td>
</tr>
<tr>
<td>Log Price 3</td>
<td>-0.199</td>
<td>0.546</td>
<td>-1.518</td>
<td>0.358</td>
</tr>
<tr>
<td>Log Price 4</td>
<td>-0.217</td>
<td>0.438</td>
<td>-1.152</td>
<td>0.140</td>
</tr>
<tr>
<td>Log Price 5</td>
<td>-0.260</td>
<td>0.568</td>
<td>-1.439</td>
<td>0.315</td>
</tr>
<tr>
<td>Log Price 6</td>
<td>-0.397</td>
<td>0.886</td>
<td>-2.088</td>
<td>0.750</td>
</tr>
<tr>
<td>Log Price 7</td>
<td>-0.254</td>
<td>0.548</td>
<td>-1.365</td>
<td>0.306</td>
</tr>
<tr>
<td>Log Price 8</td>
<td>-0.337</td>
<td>0.793</td>
<td>-1.847</td>
<td>0.632</td>
</tr>
<tr>
<td>Persons in the household</td>
<td>2.797</td>
<td>1.467</td>
<td>1.000</td>
<td>11.000</td>
</tr>
<tr>
<td>Adults in the household</td>
<td>1.950</td>
<td>0.748</td>
<td>1.000</td>
<td>7.000</td>
</tr>
<tr>
<td>Children under five years of age in the household</td>
<td>0.223</td>
<td>0.539</td>
<td>0.000</td>
<td>4.000</td>
</tr>
<tr>
<td>Unemployed persons in the household</td>
<td>0.091</td>
<td>0.320</td>
<td>0.000</td>
<td>4.000</td>
</tr>
<tr>
<td>Adults over the age of 65 years in the household</td>
<td>0.258</td>
<td>0.567</td>
<td>0.000</td>
<td>3.000</td>
</tr>
</tbody>
</table>

At this stage it is important to emphasize that in principle the model can be augmented by any number of demographic variables with different demographic
variables for each equation. As a matter of convenience in exploring demographic influences, the present system is estimated with two demographic variables for each equation, i.e. in the notation of (10), \( d_i = 2 \) for all \( i = 1, \ldots, n \). Some demographic variables are replicated over several equations. For example, in exploring a potentially non-linear effect of household size on food, accommodation and power expenditures, persons per household and the square of persons per household are employed as the two demographic variables. In other cases, notably for the categories clothing, entertainment, alcohol and miscellaneous, the influence of the gender of the household head is explored, interacting with a relevant household size variable. Table 2 summarises the demographic variables by the commodity category for which they have been allocated.

**Table 2: Commodity Categories and Demographic Variables**

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Demographic Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Food and Non-Alcoholic Beverages</td>
<td>1: number of persons in household</td>
</tr>
<tr>
<td></td>
<td>2: square of number of persons in household</td>
</tr>
<tr>
<td>2: Accommodation</td>
<td>1: number of persons in household</td>
</tr>
<tr>
<td></td>
<td>2: square of number of persons in household</td>
</tr>
<tr>
<td>3: Power</td>
<td>1: number of persons in household</td>
</tr>
<tr>
<td></td>
<td>2: square of number of persons in household</td>
</tr>
<tr>
<td>4: Clothing and Footwear</td>
<td>1: number of persons in household</td>
</tr>
<tr>
<td></td>
<td>2: persons x gender of head of household</td>
</tr>
<tr>
<td>5: Transport</td>
<td>1: number of adults in the household</td>
</tr>
<tr>
<td></td>
<td>2: number of unemployed in household</td>
</tr>
<tr>
<td>6: Medical and Personal Care</td>
<td>1: children aged under five in household</td>
</tr>
<tr>
<td></td>
<td>2: adults aged over 65 in household</td>
</tr>
<tr>
<td>7: Entertainment</td>
<td>1: number of adults in the household</td>
</tr>
<tr>
<td></td>
<td>2: adults x gender of head of household</td>
</tr>
<tr>
<td>8: Alcohol and Tobacco</td>
<td>1: number of adults in the household</td>
</tr>
<tr>
<td></td>
<td>2: adults x gender of head of household</td>
</tr>
<tr>
<td>9: Miscellaneous</td>
<td>1: number of adults in household</td>
</tr>
<tr>
<td></td>
<td>2: number of unemployed in household</td>
</tr>
</tbody>
</table>
V Estimation and Results

For econometric estimation we append additive errors to the system (10) and classical distributional assumptions are made. One equation is redundant by adding up. The system is estimated as a system of 8 non-linear equations using the non-linear maximum likelihood routine in SHAZAM Version 9.0. To satisfy adding up restrictions, \( \alpha_i \) and \( \beta_i \) are estimated residually in accordance with (3). In our current work, given that we have a system of 8 equations to estimate, we simplify slightly, and improve prospects for regularity at the expense of some flexibility, by ignoring the contribution to second order price effects coming from the quadratic term in the translog price specification. Hence we set \( \gamma_{ik} = 0 \) for all \( i \) and \( k \). The estimating equations are then, for commodity \( i \), pooled across time \( t \) and household unit \( m \):

\[
S_{imt} = \left( \sum_{j=1}^{2} \theta_{ij} x_{ijmt} + (\beta_i - \eta \alpha_i) \right) \left( \ln c_{imt} - \sum_{k=1}^{9} \alpha_k \ln p_k - \sum_{k=1}^{9} \sum_{j=1}^{2} \theta_{kj} x_{kjmt} \ln(p_{km} / c_{mt}) \right) + \epsilon_{imt}
\]

(14)

for \( i = 1, \ldots, 8 \) with the restrictions \( \alpha_i = 1 - \sum_{k=2}^{9} \alpha_k \) and \( \beta_i = 1 - \sum_{k=2}^{9} \beta_k \) and where we assume \( \epsilon_{imt} \sim N(0, \sigma_i^2) \) and \( E(\epsilon_{imt} \epsilon_{im't}) = 0, m \neq m', t \neq t' \).

Parameter estimates, including their standard errors and associated t ratios, are given in Table 3, and associated goodness of fit, residual and test statistics appear in Table 4. The notation adopted mirrors that for the equations specified in Sections 2 and 3. Thus the MAIDS parameter is denoted as \( \eta \), standard price parameters are denoted as \( \alpha_i \) and \( \beta_i \), with the commodity subscript \( i \) \( \{i=1, \ldots, 9\} \), and parameters associated with the demographic variables are denoted \( \theta_{ij} \) (i.e. commodity subscript \( i \), and
demographic variable number $j$, $\{j=1,2\}$. For example $\theta_{62}$ indicates the second demographic variable for budget share equation six (i.e. the number of adults aged over 65 in the household as a demographic explanator of the proportion of the budget spent on medical and personal care).

Table 3: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-stat</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.7240</td>
<td>0.0011</td>
<td>630.23</td>
<td>$\beta_1$</td>
<td>0.1742</td>
<td>0.0017</td>
<td>99.95</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.1897</td>
<td>0.0022</td>
<td>84.51</td>
<td>$\beta_2$</td>
<td>0.3191</td>
<td>0.0026</td>
<td>120.53</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.3376</td>
<td>0.0034</td>
<td>99.53</td>
<td>$\beta_3$</td>
<td>0.0375</td>
<td>0.0005</td>
<td>76.24</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.0472</td>
<td>0.0006</td>
<td>77.44</td>
<td>$\beta_4$</td>
<td>0.0431</td>
<td>0.0009</td>
<td>49.22</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.0378</td>
<td>0.0011</td>
<td>35.18</td>
<td>$\beta_5$</td>
<td>0.1173</td>
<td>0.0022</td>
<td>53.78</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>0.0952</td>
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<td>0.0021</td>
<td>57.85</td>
<td>$\beta_8$</td>
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<td>0.0011</td>
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<td>$\beta_9$</td>
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<td>$\theta_{11}$</td>
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<td>0.0005</td>
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<td>$\theta_{21}$</td>
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<td>-1.74</td>
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<td>$\theta_{14}$</td>
<td>-0.0020</td>
<td>0.0010</td>
<td>-1.97</td>
<td>$\theta_{24}$</td>
<td>0.0118</td>
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<td>$\theta_{26}$</td>
<td>0.0106</td>
<td>0.0007</td>
<td>15.07</td>
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<td>$\theta_{17}$</td>
<td>0.0011</td>
<td>0.0010</td>
<td>1.08</td>
<td>$\theta_{27}$</td>
<td>-0.0073</td>
<td>0.0022</td>
<td>-3.29</td>
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</table>

Table 4: Goodness of Fit, Residual Statistics and General Test Statistics

<table>
<thead>
<tr>
<th>Equation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>$R^2$</td>
<td>0.167</td>
<td>0.034</td>
<td>0.207</td>
<td>0.012</td>
<td>0.036</td>
<td>0.024</td>
<td>0.049</td>
<td>0.017</td>
<td>0.022</td>
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<tr>
<td>$\sigma_i^2$</td>
<td>0.0082</td>
<td>0.0020</td>
<td>0.0007</td>
<td>0.0049</td>
<td>0.0170</td>
<td>0.0033</td>
<td>0.0100</td>
<td>0.0041</td>
<td>0.0065</td>
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</tbody>
</table>

Log Likelihood 325,506.9

Likelihood Ratio Statistic$[\theta_{ij}=0 \text{ if } i=1,\ldots,9 \text{ or } j=2] = 3,442.6$ Critical $\chi^2(.01, 18 df) = 34.8$

Of particular interest is the effect of demographics on share predictions. Table 5 presents predicted shares at the reference (median expenditure and price) level for households with various demographic characteristics (i.e. expenditure and prices are set at unity via our scaling convention). Also included in the final row are the
limiting shares for the demography-independent case where real total expenditure asymptotes to infinity.

Table 5: Predicted Shares for Demographic Categories and Asymptotic Shares

<table>
<thead>
<tr>
<th>Equation:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male head</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>0.271</td>
<td>0.230</td>
<td>0.042</td>
<td>0.060</td>
<td>0.140</td>
<td>0.051</td>
<td>0.089</td>
<td>0.058</td>
<td>0.059</td>
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<tr>
<td>2</td>
<td>0.226</td>
<td>0.259</td>
<td>0.042</td>
<td>0.047</td>
<td>0.141</td>
<td>0.076</td>
<td>0.090</td>
<td>0.059</td>
<td>0.059</td>
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<tr>
<td>3</td>
<td>0.231</td>
<td>0.265</td>
<td>0.043</td>
<td>0.048</td>
<td>0.144</td>
<td>0.056</td>
<td>0.092</td>
<td>0.060</td>
<td>0.061</td>
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<tr>
<td>Female head</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td>0.249</td>
<td>0.242</td>
<td>0.042</td>
<td>0.056</td>
<td>0.168</td>
<td>0.051</td>
<td>0.096</td>
<td>0.039</td>
<td>0.058</td>
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<tr>
<td>5</td>
<td>0.213</td>
<td>0.298</td>
<td>0.045</td>
<td>0.044</td>
<td>0.121</td>
<td>0.070</td>
<td>0.104</td>
<td>0.042</td>
<td>0.063</td>
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<tr>
<td>Asymptotic Share</td>
<td>0.134</td>
<td>0.270</td>
<td>0.012</td>
<td>0.057</td>
<td>0.175</td>
<td>0.065</td>
<td>0.154</td>
<td>0.043</td>
<td>0.090</td>
</tr>
</tbody>
</table>

1. Household=4 persons, 2, adults, 2 children under 5, no adults over 65, and no unemployed.
2. Household=2 persons, no children, 2 adults over 65, and no unemployed.
3. Household=2persons, 2 adults, no children, no adults over 65 and no unemployed.
4. Household=3persons, 1adult, 2 children under 5, no adults over 65 and no unemployed.
5. Household=1 person, no children, one adult over 65 and no unemployed.

For our nine-commodity breakdown the results suggest that for a reference household (defined as a three-person household, with two adults, but none over 65 years of age, with one child, no unemployed persons and male head of household) there are three necessities and six luxuries. Figure 1 graphs the budget shares of the three necessities against the logarithm of real income. Figure 2 illustrates the graph of the budget shares for the three largest luxuries while Figure 3 provides the corresponding graphs for the three smaller luxuries. In these figures demographic variables are set for the reference household and commodity prices are set equal to those faced by the median household (i.e. unity by construction). This ‘typical’ household is chosen as a control against which to shift some demographic variables in Figures 4 and 5.
In Figure 4, all households face the same prices as the median household in the sample (i.e. unity). The typical household is as described for Figures 1-3 (i.e. a three-person household, with two adults, but none over 65 years of age, with one child, no unemployed persons and male head of household). The elderly household is a two-
person household with no children or unemployed persons, with both persons over 65 years of age.

**Figure 3: Shares for Three Minor Luxuries**

![Graph showing shares for three minor luxuries.]

**Figure 4: Shares for Medical and Personal Care - Typical and Elderly Households**

![Graph showing shares for medical and personal care.]

Figure 4 illustrates the importance of the ‘elderly’ demographic on medical and personal care expenditures. For the typical household in the Australian economy,
medical and personal care is a luxury. By contrast, for the elderly it is a necessity. The obvious social policy implications illustrate the importance of allowing the functional form of Engel curves to be estimated in a commodity specific fashion.

Another illustration of commodity-specific demographic effects, also with important social policy implications, is given in Figure 5. Figure 5 contrasts the expenditure share of the typical household with that of a single parent household comprising a single female parent, with two children, both under five years of age for commodity eight, alcohol and tobacco. Again the interpretation of the commodity as either a necessity or a luxury is seen to be demography-dependent.

**Figure 5: Shares for Alcohol and Tobacco - Typical and Single Parent Households**
VI Conclusion

The DEMAIDS model allows for different demographic effects for each commodity while still enforcing demand regularity conditions, thus increasing the efficiency of estimation by avoiding the inclusion of redundant demographic variables when estimating other theoretically consistent (or regular) demographic demand systems.

Similar to the work of Dickens, et. al. (1993), Blundell, et. al. (1993) and Pashardes (1995), DEMAIDS allows for non-linear effects of demographics upon the expenditure share demands. It also allows the budget shares to be written as a weighted average of the asymptotic expenditure share and the demographic effect, where the weight on the demographic effect varies from unity for the reference household to zero as expenditure approaches infinity. Thus it allows for demographic effects to be more severe at lower levels of expenditure but decline as expenditure grows.

Using a pooled cross section of Australian unit record and price data, the parameters of DEMAIDS are estimated and in general found to be highly significant. The budget share equations are shown to vary significantly across goods and also across demographics. The empirical results have significant policy implications, in that they suggest that whether goods are necessities or luxuries depend on demographics. For example, while medical and personal care is a luxury for most households it is a necessity for elderly Australian households. A similar reversal can be seen for single-parent households for whom alcohol and tobacco is a luxury, unlike the other Australian households for which it is a necessity. To the authors’ knowledge such reversals have rarely if ever been captured by regular demographic demand systems.
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