SCHOOL OF ECONOMICS

Discussion Paper 2003-08

**Quality, Monopoly and Efficiency: Some Refinements**

**Hugh Sibly**

ISSN 1443-8593
ISBN 1 86295 137 3

# QUALITY, MONOPOLY AND EFFICIENCY: SOME REFINEMENTS

**Hugh Sibly**
**School of Economics**
**University of Tasmania**
**GPO Box 252-85**
**Hobart Tas 7001**

**Email:** HSIBLY@POSTOFFICE.UTAS.EDU.AU
**Ph: (03) 62 262825**
**Fax: (03) 62 267587**
**(IDD code +61+3+)**

December 2003

ABSTRACT

This paper reformulates the full information monopoly model of goods choice (Spence, 1975 and Sheshinski, 1976) thereby identifies a number of important implications that have been overlooked in the literature. It is argued that past specifications of consumer preferences have been incomplete. Profit maximising and efficient outcomes are compared under the reformulation. Output is interpreted as being the number of units of quality produced, and is chosen according to familiar rules. It is shown that quality per item is systematically related to the level of quality per item that minimises cost. A realistic set of preference is identified for which the efficient quality per item is greater than the monopoly's choice.

JEL Classification: L11

# QUALITY, MONOPOLY AND EFFICIENCY: SOME REFINEMENTS

Hugh Sibly
University of Tasmania

For most consumers the quality of a good is as important an attribute as its price. However, while the analysis of price is ubiquitous in the economic literature, the quality of goods is more rarely discussed. The landmark treatments of a monopolist's choice of quality choice are provided by Spence (1975) and Sheshinski (1976). This paper reformulates their full information model, and thereby identifies a number of important implications that, to my knowledge, have been overlooked in the literature.[1] Although the industrial organization literature moved to consider more conceptually complex issues regarding pricing and quality (for example product differentiation and adverse selection), this was done at the cost of adopting simplifying assumptions (such consumers only demand one unit of the good).[2] Such simplifying assumptions, while allowing analytical progress, are not always warranted.[3] The reformulation presented in this paper allows a more complete insight into full information monopoly model. This is important in understanding the implication of the simplifications made in the literature, and indicating how these might best be relaxed in future work.

The starting point for the existing literature is an inverse demand function that is a function of number of items (noi) and quality per item (qpi). (Schmalensee, 1979). Spence and Sheshinski adopt a general demand function, only requiring that price (per item) reduce demand and qpi increase demand. The special case, in which the inverse

---

[1] As can be seen from the discussion below, this papers approach has been foreshadowed, but not followed up, in the literature.

[2] For example Mussa and Rosen (1978) and Shaked and Sutton (1983).

[3] Variation in demand per customer is a fundamental characteristic of many market, for example those supplied by public utilities

demand function depends only on the number of units of quality (uoq) consumed (ie the product of noi and qpi), is also given attention by the literature (Levari and Peles, 1973, and Kihlstrom and Levari, 1977).

In fact it is natural to consider consumer utility as a function of uoq , because qpi is embedded in the good. The uoq consumed therefore captures the actual experience of consuming that good. For example the experience of eating a particular variety of cheese is given by the product of the experience (quality) of each taste and the number of tastes. In addition, however, consumers may also have an autonomous taste for qpi. That is, additional utility arising from the qpi that is independent of the uoq consumed. For example, if consumers gained utility from knowing they were eating a high quality cheese, in addition to the utility gained from uoq consumed, they would have an 'autonomous taste for qpi'.

In this paper it is assumed that utility is a function of uoq and qpi. It is shown that when a consumer has an autonomous taste for qpi two important sub-cases can be distinguished: when the consumer does and does not have an autonomous demand for qpi. A consumer has (does not have) an autonomous demand for qpi when, holding price of quality constant, an increase in qpi increases (does not change) demand for uoq.

The case in which consumers have an autonomous taste for qpi but do not have an autonomous demand for qpi has been overlooked in the existing literature. However such preferences may be a realistic description of consumers, particularly those of a public utility. An example is urban water supply. In her use of water, the consumer cares not only about the quantity of water consumed but the pressure (qpi) at which this water is supplied. The consumer's utility therefore is a function of uoq. In addition, the consumer may value the existence of a high pressure water supply for potential fire fighting purposes. The consumer gains utility from the existence of a high quality supply, irrespective of the uoq used. This gives rise to the consumer's autonomous taste for qpi. However an increase in the qpi (pressure), while holding the price of quality fixed, does not increase the uoq consumed. Numerous other examples, in which consumers have an autonomous taste, but no autonomous demand, for qpi could be cited.

Given the above formulation of consumer behaviour, it is natural to think of a monopoly choosing its profit maximizing uoq and qpi, rather than the noi and qpi. Cost can be written as a function of uoq and qpi. The uoq can be treated as the output of the firm, and it is chosen according to the familiar rules. When consumers do not have an autonomous demand for qpi (ie inverse demand is independent of qpi) the monopoly chooses qpi to minimize the cost of producing the monopoly number of uoq (Levari and Peles, 1973).[4] When consumers do have an autonomous demand for qpi, an increase in qpi increases revenue. The monopoly therefore has an incentive to provide a level of qpi greater than the cost minimising level.

A similar argument applies to the choice of efficient uoq and qpi. The efficient uoq is that level which equates the price of quality with the marginal cost of uoq. When consumers do not have an autonomous taste for qpi, the efficient qpi is the one that minimises cost, given the efficient uoq is produced. However when consumers do have an autonomous taste for qpi, there is an additional social benefit in producing qpi. In this case the efficient qpi is greater than the cost minimising qpi.

This approach therefore highlights the important role of the cost minimising level of qpi. To understand the relationship between the efficient and monopoly levels of qpi it is therefore necessary to describe how the cost minimizing level of qpi varies with uoq. The cost minimizing level of qpi is independent of the number of uoq when the cost function exhibits ‘constant returns to scale’. This assumption on the cost function is often reasonable when qpi is rival. This conclusion is known as the ‘Swan independence result’. On the other hand when cost is additively separable, and marginal cost is increasing in both qpi and quantity, the cost minimizing level of qpi increases with noi. This assumption is often warranted when qpi is non-rival.

From the above arguments it can be concluded that, if consumers have a taste, but no demand, for qpi then the efficient qpi is greater the monopoly qpi when cost exhibits either constant returns to scale or is additively separable. This result contrasts with the case in consumers have both an autonomous taste and an autonomous demand for qpi. In

---

[4] There exists a cost minimizing level of qpi because an increase in qpi has two effects on cost. One is the direct cost of producing increased qpi. The second effect is that the quantity required to produce a given number of uoq is reduced, thereby reducing cost. Cost is minimized at the qpi where the former effect just outweighs the latter effect.

this case the efficient qpi may be either more or less than the monopoly qpi. This ambiguity corresponds to the findings Spence and Sheshinski.

The approach of this paper, however, suggests that the sign of marginal consumer surplus with respect to qpi is the natural indicator of relative size of the efficient and monopoly qpi. This contrasts with the approach of Spence and Sheshinski, which emphasises the sign of the cross partial derivative of the inverse demand function in creating the indeterminacy. It is shown that the use of the sign partial derivative of the inverse demand function may not be correct when consumers have an autonomous taste for qpi. Further, information on the inverse demand curve is required for inframarginal consumers, where information on the marginal consumer surplus with respect to qpi, is required only for the marginal consumer. Overall, therefore, the analysis of this paper suggests that the marginal consumer surplus with respect to qpi may be a useful guide to regulators when assessing qpi levels.

This paper applies the above approach to model a regulated public utility that must negotiate with a regulator over its quantity and quality of production. As is the case in many jurisdictions, the regulator is assumed to be a consumer advocate. On this assumption the optimal bargain is derived. It is shown that when consumers do not have an autonomous taste for qpi, firms minimises the cost of producing the bargained uoq. On the other hand, when consumers have an autonomous taste for qpi, the bargained qpi is greater than the cost minimising level.

Section 1 of the paper introduces the model. The paper's analysis begins, in section 1.1, with a derivation of both individual and total demand. Autonomous taste and demand for qpi is defined. Section 1.2 analyses the costs facing the firm. Expressions for the cost minimising qpi are obtained. Section 2.1 derives the monopolists choice of qpi.. The efficient choice of qpi is found in section 2.2. The efficient and monopoly qpi are compared. The model of a regulated public utility, in which the public utility and the regulator bargain over the firm's quality and quantity, is presented in section 3. This analysis is used to predict the impact of a change in the regulators power on qpi. Section 4 concludes the paper.

# 1. The Model

## 1.1 Consumers

Following the argument in the introduction, assume consumer i's utility function is may be represented in the form $U^i(qX^i, q, \underline{\mathbf{x}}^i)$, where i=1…k, $X^i$ is the number of items consumed by consumer i, $\underline{\mathbf{x}}^i$ is a vector of the consumption of other goods by consumer i and q is the qpi of the good as perceived by customers or 'perceived qpi'. The perceived qpi is common to all consumers and is exogenous from the point of view of the individual consumer. A consumers' utility function said to be 'separable in qpi' if it can be represented in the form $U^i(qX^i, q, \underline{\mathbf{x}}^i) = \Theta^i(q)Y^i(qX^i, \underline{\mathbf{x}}^i)$. Such a form may be plausible in many instances because, as noted in the introduction, a benefit from qpi may be independent of the uoq consumed.

Define $x^i$, consumer i's demand for uoq, by $x^i \equiv qX^i$. The consumer's budget constraint is:

$$PX^i + \underline{\mathbf{p}}.\underline{\mathbf{x}}^i = I \qquad (1)$$

where P is the price of an item, and $\underline{\mathbf{p}}$ it the vector of prices of other goods. Define p, the price of quality, by p =P/q. Then the consumer i's optimisation problem becomes:

$$\text{Max } U^i(x^i, q, \underline{\mathbf{x}}^i) \text{ subject to } px^i + \underline{\mathbf{p}}.\underline{\mathbf{x}}^i = I \qquad (2)$$

Using standard consumer theory the demand for uoq and demand for noi is given by:

$$x^i = x^i(p, q, \underline{\mathbf{p}}, I) \quad \Leftrightarrow \quad X^i(P, q, \underline{\mathbf{p}}, I) = x^i(P/q, q, \underline{\mathbf{p}}, I)/q \qquad (3)$$

The demand for uoq suggests the following definition:

Definition 1: Consumer i is said to have an autonomous demand for qpi if $x_2^i(p, q, \underline{\mathbf{p}}, I)>0$ and not to have an autonomous demand for qpi if $x_2^i(p, q, \underline{\mathbf{p}}, I)=0$.[5]

---

[5] In this paper the subscript k represents the derivative of the function. Thus $f_k$ represents the derivate with respect to the $k^{th}$ argument of the function f . All functions are assumed to be appropriately differentiable.

Consumers do not have an autonomous demand for qpi if (i) consumer i's utility is independent of qpi, ie $U^i_2=0$, or (ii) the utility function is separable in qpi. As both **p** and I are exogenous to the analysis below, for brevity subsequent reference to them is suppressed. Define total demand for uoq, x(p,q), and total demand for noi, X(P,q) as:

$$x(p,q) = \sum_{i=1}^{k} x^i(p,q) \qquad \Leftrightarrow \qquad X(P,q) = \sum_{i=1}^{k} X^i(P,q) \qquad (4)$$

If all consumers do not have an autonomous demand for qpi then $x_2(p,q)=0$. The inverse demand for uoq, p(x,q), can be obtained by inverting (4). Note that:

$$p_2(x,q) = -x_2(p,q)/x_1(p,q) \qquad (5)$$

so that $p_2(x,q)$ >(=) 0 if consumers do (do not) have an autonomous demand for qpi. The inverse demand for noi, P(X,q) can similarly be obtained from (4). The literature typically assume $P_2(X,q)>0$ as the starting point of its analysis. But observe that:

$$p_2(x,q) = -xP_1(x,q)/q^2 + P_2(X,q) \qquad (6)$$

If $P_2>0$ the consumer has an autonomous demand for qpi. However the converse is not necessarily true. The assumption of an autonomous demand for qpi is therefore more restrictive than the usual assumption, $P_2>0$, made in the literature. The following proposition is readily established from (3) and (4):

Proposition 1:The elasticity of noi demanded with respect to price per item, $\varepsilon_{XP}$, is related to $\varepsilon_x$, the elasticity of uoq with respect to the price of quality (the elasticity of demand) in the following way:

$$\varepsilon_{XP} \equiv (P/X)(\partial X/\partial P) = px_1(p,q)/x \equiv -\varepsilon_x \qquad (7)$$

The elasticity of noi demanded with respect to quality, $\varepsilon_{Xq}$, is:

$$\varepsilon_{Xq} \equiv (q/X)(\partial X/\partial q) = \varepsilon_{xq} + \varepsilon_x \; -1 \qquad (8)$$

where $\varepsilon_{xq} \equiv qx_2(p,q)/x$ is the elasticity of uoq with respect to qpi.

The impact of an increase in qpi on demand for noi is the sum of two effects. First, the consumers' autonomous taste for qpi causes an increase in the demand for uoq. Second, the increase in perceived qpi reduces the price of quality, thus increasing the demand for uoq. As the demand for uoq is the product of noi and qpi, this effect reduces (increases) noi if the elasticity of demand, $\varepsilon_x$, is less (more) than one. Overall, an increase in perceived qpi reduces (increases) noi if the sum of the elasticity of uoq with respect to autonomous qpi and the elasticity of demand, $(\varepsilon_{xq} + \varepsilon_x)$, is less than 1.

The industrial organization literature generally formulates its analysis using consumer surplus and consumer benefit rather than consumer utility. For consistency with this approach, and also for ease of analysis, this approach is adopted in this paper. The following result is therefore required.

<u>Proposition 2</u>: When consumers have an autonomous demand for qpi, total consumer surplus, v(p,q), may be expressed in the following way:

$$v(p,q) = w(p,q) + \varpi(q) \qquad (9)$$

where $v_1(p,q) = w_1(p,q)<0$ and $w_2(p,q)<0$.When consumers have no autonomous demand for uoq consumer surplus is additively separable:

$$v(p,q) = \omega(p) + \varpi(q) \qquad (10)$$

where $v_1(p,q) = \omega'(p)<0$.

Proof: Integrating consumer i's uoq demand curve yields their consumer surplus[6], $v^i(p,q)$, where:

$$v^i(p,q) = w^i(p,q) + \varpi^i(q) \qquad (11)$$

[6] The surplus provides a good approximation of welfare when income effects are negligible, in particular when $PX^i$ is a small fraction of the consumer's income (see Tirole, 1988, p. 11).

and where $w^i(p,q) = \int_p^{\infty} x^i(r,q)dr$ is the consumption dependent, and $\varpi^i(q)$ the autonomous, component of consumer surplus.[7] Note

$$w_2^i(p,q) = \int_p^{\infty} x_2^i(r,q)dq > 0 \qquad (12)$$

if consumer i has an autonomous demand for qpi. Total consumer surplus, v(p,q), is given by

$$v(p,q) = \sum_{k=1}^{n} v^i(p,q) = \sum_{k=1}^{n} [w^i(p,q) + \varpi^i(q)] = w(p,q) + \varpi(q) \qquad (13)$$

Note that $v_1(p,q)= w_1(p,q)=-x<0$ and therefore $\partial v/\partial P=-X$. (12) shows $w_2>0$. When consumer i has no autonomous demand for qpi:

$$v^i(p,q) = \omega^i(p) + \varpi^i(q) \qquad (14)$$

where $\omega^i(p) = \int_p^{\infty} x^i(r)dr$. Thus when a consumer has no autonomous demand for qpi their consumer surplus is additively separable. Total consumer surplus is given by:

$$v(p,q) = \sum_{k=1}^{n} v^i(p,q) = \sum_{k=1}^{n} [\omega^i(p) + \varpi^i(q)] = \omega(p) + \varpi(q) \qquad (15) \quad ||$$

Proposition 2 lead to the following:

Proposition 3: Consumer surplus, V(x,q) may be expressed as function of total uoq and qpi in the following way:

$$V(x,q) \equiv W(x,q) + \varpi(q) \qquad (16)$$

---

[7] $\varpi^i(q)$ appears in (11) as a constant of integration. Sheshinksi p.127 notes the possibility of this constant, but omits the possibility it may depend on qpi.

where $V(x,y)= v(p(x,q),q)$, $W(x,y) = w(p(x,q),q)$, $W(0,q)=0$ and $V_1=W_1>0$. The sign of $W_2$ (and hence $V_2$) is ambiguous. When consumers have no autonomous demand for uoq consumer surplus is additively separable:

$$V(x,q) \equiv \Omega(x) + \varpi(q) \tag{17}$$

where $\Omega(x) = \omega(p(x))$ and thus $V_1 = \Omega'(x) >0$.

Proof: Total consumer surplus may be expressed as a function of uoq and qpi by substitution of the inverse total demand for uoq function yields (16). Note that $V_1(x,q) = v_1(p,q).p_1(x,q) = -p_1(x,q)x = p/\varepsilon_x > 0$. Further $V_2(x,q)= v_1(p,q).p_2(x,q) + v_2(p,q) = -xp_2(x,q) + w_2(p,q) + \varpi'(q)$. The sign of $V_2$ is therefore ambiguous. When consumers have no autonomous demand for qpi V is additively separable:

$$V(x,q) = \Omega(x) + \varpi(q) \tag{18}$$

where $\Omega(x) = \omega(p(x))$. Further $V_2(x,q)= \varpi'(q)>0$. ||

$\varpi(q)$ is called autonomous consumer surplus. It can be interpreted as the component of consumer surplus that is independent of the level of consumption. The non-autonomous component of consumer surplus, $W(x,q)$, measures the area under the demand curve, $x(p,q)$, over the interval $[p(x),\infty)$. (It is therefore equivalent to the 'usual' measure when the good's only property is quantity consumed.) The impact of an increase of qpi on $W(x,q)$ is ambiguous for the following reason. An increase in qpi shifts the demand for uoq curve upward, increasing the surplus the consumer gains from consuming the given uoq. However the price of quality must also increase to keep uoq constant following the increase in qpi, thus lowering consumer surplus. The overall effect is ambiguous. When, however, consumers have no autonomous demand for qpi then an increase in quality does not shift the demand for uoq curve and thus does not increase price of quality. Consumer surplus changes only because its autonomous component, $\varpi(q)$, changes.

Let $B(x,q)=V(x,q)+p(x,q)x$ be consumer benefit. Observe that:

$$B_1(x,q) = p(x,q) >0 \tag{19}$$

and:

$$B_2(x,q)= v_2(p,q) = w_2(p,q) + \varpi'(q) \tag{20}$$

This definition of consumer benefit is used to describe consumer preferences.

<u>Definition 2</u>: Consumers are said to have an autonomous taste for qpi if $B_2(x,q)>0$ for all $x\geq 0$ and $q\geq 0$ .

The following proposition uses this definition.

<u>Proposition 4</u>: If consumers have an autonomous taste for qpi then $\varpi'(q)\geq 0$.

Proof: As $B_2(x,q) = W_2(x,q) + \varpi'(q) + p_2(x,q)x$, therefore $B_2(0,q) = \varpi'(q)$. If $B_2(0,q)>0$ for all $q\geq 0$ then $\varpi'(q)>0$ for all $q\geq 0$. ||

<u>Proposition 5</u>: If consumers have an autonomous demand for qpi and $\varpi'(q)\geq 0$, then they have an autonomous taste for qpi. However the reverse is not true.

Proof: If consumers have an autonomous demand for qpi then $w_2>0$ and thus, by (20) they have an autonomous taste for qpi. However (17) shows that consumers can have an autonomous taste for qpi without having an autonomous demand for qpi. ||

It is technically possible for consumers to have both an autonomous demand for qpi and also for $\varpi'(q)<0$. In this case the sign of $B_2$ would be ambiguous, and, if $B_2$ were negative, consumers would not have an autonomous taste for qpi. However the possibility that both $x_2>0$ and $\varpi'(q)<0$ appears unrealistic, so is ruled out below.

Observe that (19) implies $B_{12}(x,q) = p_2(x,q)$. Using this result the following proposition can be established.

<u>Proposition 6</u>: Suppose consumers have an autonomous taste for qpi. If $p_{21}(\xi,q)<0$ for all $0\leq\xi\leq x$ then $V_2(x,q)>0$. However $p_{21}(\xi,q)>0$ for all $0\leq\xi\leq x$ does not guarantee that $V_2(x,q)<0$.

Proof: Observe

$$B_2(x,q) = \int_0^x B_{21}(\xi,q)d\xi + \varpi'(q) = \int_0^x p_2(\xi,q)d\xi + \varpi'(q) \qquad (21)$$

Thus:

$$V_2 = B_2(x,q) - p_2(x,q)x = \left[\int_0^x p_2(\xi,q)d\xi - p_2(x,q)x\right] + \varpi'(q) \qquad (22)$$

If $p_{21}<0$ for all $x\geq0$ and $q\geq0$ then the expression in the square brackets in (22) is positive, and therefore $V_2$ is positive. On the other hand if $p_{21}>0$ for all $x\geq0$ and $q\geq0$ then the expression in square brackets is negative and the sign of $V_2$ is ambiguous. ||

Proposition 6 is presented to motivate the approach used below. In Spence and Sheshinki's papers the cross derivative of the inverse demand curve, $P_{21}(X,q)$, was given a central role in assessing the difference between monopolistic and efficient qpi. However their formulation excluded the possibility of an autonomous consumer surplus (ie they assume $\varpi(q)=0$). They therefore assume that $P_{21}(X,q)>0$ implies an increase in qpi reduces consumer surplus. Proposition 6 shows this is not the case when autonomous consumer surplus is allowed for. When $p_{21}>0$, an increase in qpi reduces the area of consumer surplus under the demand curve (ie W(x,q)) but this may be offset by an increase in autonomous consumer surplus.

Another reason to reject the use of $p_{21}$ is that, to be useful, it must have a given sign over the range $0\leq\xi\leq x$. Many realistic demand curves do not satisfy this criterion. Furthermore these inframarginal values of $p_{21}$ are never observed in practice.

## 1.2 Technology

Perceived qpi is related to technical qpi, which is an objective measure of the quality level of the good. Allowing for a distinction between the two concepts of quality admits more general utility function to the analysis. For instance, it allows the possibility that consumers exhibit diminishing returns to technical qpi in their perception of qpi.[8] Perceived qpi is assumed to increase with technical qpi. Let y(q) be the technical qpi required to achieve the perceived qpi q, where $y'\geq0$. As the analysis of firm's production decision is conducted in terms of perceived qpi, for brevity it is referred to below as simply qpi.

---

[8] By judicious definition of perceived qpi, a wide class of consumer preferences are admitted to the analysis. For example, if benefit is $y^3(1+X\ln y)$, the approach of this paper is followed by setting $q = \ln y$. One consequence of admitting such transformations is that there may be no unique measure of perceived qpi.

Firms technology is summarised by the cost function C(X, y), ie the total cost of production is a function of the number of units produced and the technical qpi. It is assumed that marginal cost, $C_1(X,y)$, and the marginal cost of technical qpi, $C_2(X,y)$, are non-decreasing, ie $C_1 \geq 0$ and $C_2 \geq 0$.

By (3) and (4) X=x/q. Therefore the cost function can be expressed as a function of uoq produced and qpi, c(x,q), in the following way:

$$c(x,q) \equiv C(x/q, y(q)) \qquad (23)$$

If the uoq produced is considered fixed, c(x,y) represents as the cost of producing x uoq when qpi is varied. The following optimisation problem:

$$\min_{q} \; c(x,q) \qquad (24)$$

determines the minimum cost of creating x uoq. The first order condition of (24) is:

$$c_2(x,q) = -XC_1(X,y)/q + C_2(X,y)y'(q) = 0 \qquad (25)$$

or:

$$yC_2(X,y)\varepsilon_{yq} = XC_1(X,y) \qquad (26)$$

where $\varepsilon_{yq} \equiv qy'/y$ is the elasticity of technical qpi with respect to (perceived) qpi. Equation (25) defines the cost minimising qpi, q*(x) for a given uoq, while equation (26) defines the cost minimising technical qpi, y*(X) for a given noi. Figure 1 depicts the marginal cost of quality, $c_2(x,q)$, as a function of qpi for a given uoq. The marginal cost of qpi is upward sloping as it is assumed that $c_{22}>0$. $q^*(x)$ occurs when the curve cuts the horizontal axis. As can be seen from figure 1, if $c_2<(>)0$ qpi is less (more) than q*(x).

The cost function summaries not only the technology of the firm, but also the technology of use. For example, qpi can have either a rival or non-rival nature. Examples of the first type of good are roads (indeed infrastructure in general). An improvement in the quality per mile of a road automatically improves qpi for each consumer, irrespective of use level. An example of the second type of good is a car. The improvement in quality of one car does not automatically raise the quality of other cars.

Goods for which qpi is non-rival are captured by assuming the cost function is additively separable:

$$c(X,y) = \Phi(X) + \Psi(y) \quad (27)$$

where $\Phi'(X)>0$, $\Psi'(y)>0$, $\Phi''(X)>0$ and $\Psi''(y)>0$. The cost associated with qpi is independent of noi. In this case the cost minimising qpi satisfies, (26), satisfies:

$$y\Psi'(y)\varepsilon_{yq}(y) = X\Phi'(X) \quad (28)$$

If $\varepsilon_{yq}(y)$ is a non-decreasing function, then differentiation of (28) demonstrates that an increase in output increases the cost minimising technical qpi. Further, when cost is additively separable, $q^*(x)$ increases with x.

Goods in which qpi is rival may be captured by the assumption of multiplicatively separable costs:

$$c(X,y) = \chi(X)\psi(y) \quad (29)$$

where $\chi'(X)>0$ and $\psi'(y)>0$. The case in which qpi is rival is best illustrated where $\chi(X)=X$ and $\psi(y) = \omega+\zeta(y)$, where w represents (constant) marginal cost and $\zeta(y)$ the cost of qpi. If (29) holds, the cost minimising qpi satisfies:

$$\frac{y\psi'(y)\varepsilon_{yq}(y)}{\psi(y)} = \frac{X\chi'(X)}{\chi(X)} \Leftrightarrow \varepsilon_{\psi y}(y)\varepsilon_{yq}(y)= \varepsilon_{\chi X}(X) \quad (30)$$

where $\varepsilon_{\chi X}(X)\equiv X\chi'(X)/\chi(X)$ and $\varepsilon_{\psi y}(y)=y\psi'(y)/\psi(y)$. Observe that the cost minimising technical qpi, $y^*$, is independent of noi if the RHS of (30) is independent of X. Note that this can only occur if $\varepsilon_{\chi X}(X)$ is isoelastic, that is. $\varepsilon_{\chi X}(X)=\chi$, where $\chi$ is a constant. In this event the cost minimising qpi is given by:

$$\varepsilon_{\psi y}(y^*)\varepsilon_{yq}(y^*) = \chi \quad (31)$$

Note that if the cost minimising technical qpi is independent of X, then the cost minimising qpi, $q^*$, is also independent of x.

# 2. Monopoly and Efficiency

This section considers and compares the monopolistic and efficient qpi under the assumptions outlined in the previous section.

## 2.1 Monopoly

The profit of the firm, $\pi(x,q)$ , is given by:

$$\pi(x,q) \equiv p(x,q)x - c(x,q) = PX - C(X,y) \qquad (32)$$

The firm chooses uoq and qpi to maximise profits. The first order condition, $\pi_1(x,y)=0$, yields the following condition for, $x^m(q)$, the profit maximising uoq:

$$p(x,q) = \frac{c_1(x,q).\ \varepsilon_x}{(\varepsilon_x\text{-}1)} \Leftrightarrow P(X,y) = \frac{C_1(X,y).\ \varepsilon_x}{(\varepsilon_x\text{-}1)} \qquad (33)$$

This is the usual condition for the choice of profit maximising output. The first order condition, $\pi_2(x,y)=0$, yields:

$$c_2(x,q) = p_2(x,q)x \geq 0 \qquad (34)$$

Equation (34) defines $q^m(x)$, the profit maximising qpi given the uoq produced. This equation yields:

<u>Proposition 7</u>: When consumers do not have an autonomous demand for qpi ($p_2$=0) the monopolist chooses qpi to minimises the cost of producing the monopoly uoq. However when consumers do have an autonomous demand for qpi ($p_2$>0) the monopolist chooses a qpi greater than that which minimises the cost of producing the monopoly uoq.

The case in which consumers have an autonomous demand for qpi is depicted in figure 1, which indicates that for every uoq produced (including the monopoly level) the profit maximising level qpi, $q^m(x)$, is greater than $q^*(x)$, the cost minimising qpi. Intuitively, if consumers have an autonomous taste for qpi the firm can increase the elasticity of demand by increasing qpi. It is therefore profit maximising for the monopolist to increase qpi beyond the cost minimising level.

The profit maximising levels of uoq and qpi, $x^m$ and $q^m$ ($=q^m(x^m)$) simultaneously satisfy (33) and (34). These conditions yield the following:

Proposition 8: The firm's cost revenue ratio is given by:

$$\frac{C}{PX} = \frac{\varepsilon_x + \varepsilon_{xq} - 1}{\varepsilon_x \varepsilon_{cy} \varepsilon_{yq}} \qquad (35)$$

where $\varepsilon_{cy} \equiv yC_2(X,y)/C(X,y)$.

Proof: (35) is obtained by substituting (34) into (33), and then using (5) ||

Condition (35) is a generalisation of the Dorfman-Stiener condition. It indicates that an increase in the elasticity of the demand for uoq with respect to qpi increases the cost revenue ratio. Intuitively an increase in this elasticity induces the firm to produce a higher qpi thus increase the relative size of costs. The Dorfman-Stiener condition is usually presented in following form, which can be obtained by substituting (8) into (35):

$$\frac{C}{PX} = \frac{\varepsilon_{Xq}}{\varepsilon_x \varepsilon_{cy} \varepsilon_{yq}} \qquad (36)$$

Traditionally, the Dorfman-Stiener condition is used to consider the independent impact of $\varepsilon_x$ and $\varepsilon_{Xq}$ on the cost revenue ratio. Thus an increase in the elasticity of demand is predicted to reduce the cost revenue ratio. Intuitively, an increase in the elasticity of demand allows the firm to increase its price for every level of qpi. However, as shown by (8), there is a relationship between $\varepsilon_x$ and $\varepsilon_{Xq}$. An increase in the elasticity of demand also increases the elasticity of noi with respect to qpi, and this causes the firm to increase qpi. Therefore cost increases. Indeed, as shown in (35), this second effect dominates when $\varepsilon_{Xq}>1$. In this case an increase in the elasticity of demand increases the cost revenue ratio.

## 2.2. Efficient quantity and quality

The social surplus, S(x,q), is given by the sum of consumer surplus and profit:

$$S(x,q) = V(x,q) + p(x,q)x - c(x,q) = B(x,q) - c(x,q) \qquad (37)$$

The efficient uoq and qpi maximise the social surplus. The first order condition, $S_1(x,q)=0$, yields:

$$p(x,q) = c_1(x,q) \Leftrightarrow P(X,y) = C_1(X,y) \qquad (38)$$

Equation (38) defines $x^e(q)$ the efficient uoq given the level of qpi. It is the usual condition for producing the efficient level of output: efficiency requires uoq to be produced until the price of quality equals $c_1$, marginal cost. The first order condition, given by $S_2(x,q)=0$, yields:

$$c_2(x,q) = B_2(x,q) = V_2(x,q)+ p_2(x,q)x \geq 0 \qquad (39)$$

Equation (39) defines $q^e(x)$, the efficient level of qpi given the production of uoq. The efficient qpi is that level for which the marginal benefit of qpi is equal to the marginal cost of qpi. The following proposition follows from (39):

> Proposition 9: When consumers do not have an autonomous taste for qpi ($B_2=0$), then $q^e(x)=q^*(x)$. However, when consumers do have an autonomous taste for qpi (and thus $B_2>0$), then $q^e(x)>q^*(x)$.

The following proposition follows from comparison of (39) with (34))

> Proposition 10: $q^e(x)$ >(<) $q^m(x)$ if $V_2(x,q)$ >(<) 0.

The determination of $q^e(x)$ is shown in figure 1 as the intersection of $c_2(x,q)$ and $B_2(x,q)$. As $B_2>0$ in figure 1, $q^e(x)$ is greater than $q^*(x)$. Note that figure 1 is drawn on the assumption $V_2>0$ and hence $B_2>p_2x$. Under this assumption it is shown that $q^e(x)$ must be greater than $q^m(x)$. However if $V_2<0$ it is readily determined that $q^e(x)$ must be less than $q^m(x)$.

The efficient uoq, $x^e$, and efficient qpi, $q^e$, simultaneously satisfy (39) and (38), ie $x^e=x^e(q^e)$ and $q^e=q^e(x^e)$). It is of interest to know whether $q^e>q^m$. From Proposition 10 it is tempting to conclude that the efficient level of quality is greater (less) than the monopoly level of quality when $V_2>0$ ($V_2<0$). However such reasoning ignores the manner in which the functions in (39) change as x changes.

Proposition 11:If $V_2(x^m,q) > 0$ and $c_{21} \leq 0$, then $q^e > q^m$ .

Proof: From Proposition 10 $q^e(x^m)>(<)q^m$ when $V_2(x^m,y)>(<) 0$. If $q^e(x)$ increases with x then $q^e(x^e)> q^e(x^m)$. Differentiating (39) yields:

$$\frac{dq^e(x)}{dx} = \frac{p_2 - c_{12}}{c_{22} - V_{22} - p_{22}x} \qquad (40)$$

The second order conditions require that the denominator of (40) be positive. (In terms of figure 1, it requires that the curve $c_2$ have a greater slope than $B_2$.) Thus the sign of $dq^e/dx$ depends on the sign of the numerator of (40). As $p_2>0$, the numerator of (40) is positive if $c_{12} \leq 0$. ||

The overall impact of an increase of x on $q^e(x)$ can be interpreted using figure 1. An increase in x causes $B_2$ to shift by the amount $B_{21}$. As $B_{21}=p_2 \geq 0$, an increase in x causes the marginal benefit of qpi curve to unambiguously shift upward when consumers have an autonomous demand for qpi. The impact of an increase in x on $q^e(x)$ is therefore determined by the sign of $c_{21}$. If $c_{21} \leq 0$, $c_2$ either shifts rightward or remains unchanged following an increase in x, and $q^e(x)$ therefore increases. On the other hand, if $c_{12}>0$ an increase in x shifts $c_2$ upward, and the direction of change in $q^e(x)$ is ambiguous. Similarly if $V_2(x^m,q)<0$ and/or $c_{21}>0$ this analysis cannot determine whether $q^e$ is greater or less than $q^m$.

When cost additively separable, as in (27), it is readily shown that $c_{21}<0$. In this case, by Proposition 11, the efficient qpi is greater than monopoly qpi if $V_2(x^m,q) > 0$.

Now consider the case in which cost is multiplicatively separable as in (29) and $\chi(X)$ is iso-elastic. The cost minimising qpi, q*, is independent of x. Further $c_{12}>0$ for $q>q^*$, $c_{12}=0$ for $q=q^*$ and $c_{12}<0$ for $q<q^*$. (That is, as shown in figure 2, an increase in x rotates $c_2$ anti-clockwise around q*.) In this case an increase in x shifts both the marginal benefit and marginal cost curve upward, thus the sign of $dq^e/dx$ is ambiguous. However if (i) $V_2(x^m,q)>0$, and $p_2$ is larger than $c_{12}$ (and hence $dq^e/dx>0$), then $q^e>q^m$ and (ii) $V_2(x^m,q)<0$, and $p_2$ is less than $c_{12}$ (and hence $dq^e/dx<0$), then $q^e<q^m$. The magnitude of $c_{12}$, and thus the sign of $dq^e/dx$, is related to the magnitude of the elasticity of $\chi(X)$, $\chi$. For sufficiently large$\chi$, $q^e(x^e)<q^e(x^m)$. Indeed, as shown in figure 2, even if $V_2(x^m,q) > 0$, $q^e<q^m$ if $\chi$, is sufficiently large .

Now consider the case consumers have an autonomous taste for qpi but no autonomous demand for qpi . As consumers have no autonomous demand for qpi $p_2$=0, and therefore the monopoly qpi is the cost minimising qpi. Because consumers have an autonomous taste for qpi, $V_2>0$ and hence $B_2>0$. This is depicted in figure 3. For expositional purposes, the assumption that cost is multiplicatively separable with iso-elastic $\chi(X)$ is maintained. As shown in figure 3, the efficient qpi is greater than the monopoly qpi. Intuitively, an autonomous taste for qpi means it is efficient to produce a qpi greater than the cost minimising level. However if there is no autonomous demand for qpi, it is not profit maximising for the firm to produce qpi beyond the cost minimising level (as it does not increase consumers willingness to pay). By similar reasoning:

<u>Proposition 12</u>: If consumers have an autonomous taste for qpi but no autonomous demand for qpi then $q^e>q^m$ if $dq^*(x)/dx\geq 0$ .

Proposition 12's condition that $dq^*(x)/dx\geq 0$ is satisfied by both the multiplicative and additively separable cost functions.

Note that, even when consumers have an autonomous demand for qpi, $q^e>q^*(x^e)$. This limits the extent to which $q^e$ can lie below $q^m$. This is particularly the case if $dq^*(x)/dx\geq 0$ and $q^m$ is itself not much greater than $q^*(x^m)$ (as would be the case if $p_2$ was relatively small). In this event $q^m$ is not much greater than $q^*(x^e)$, and thus cannot be much greater than $q^e$.

As a practical matter, the efficient qpi might best be found sequentially, rather than by comparison with the monopoly qpi. That is, a regulator might first ensure the efficient uoq is produced. The efficient qpi is either greater (as shown in figure 1) or less than the qpi chosen by the monopoly, $q^m(x^e)$, depending on whether $V_2(x^e,q)$ is positive or negative. A regulator could use survey techniques to determine the sign of $V_2$.

# 3. A Regulated Public Utility

In reality most regulators operate in a political environment. The regulator must bargain with the public utility over its quality produced and quality levels. In the bargaining process, the public utility is concerned to maximise its profits. As is often the case, the regulator is a consumer advocate, and acts to maximise consumer surplus. This may reflect the interests of the legislators, whose electoral success may be influenced by the satisfaction of consumers.

## 3.1 The Bargaining process

The negotiation between the firm and the regulator satisfies the asymmetric Nash bargaining solution (see Eichberger, (1993, Ch 9)).[9] Under the asymmetric Nash Bargaining solution, agents negotiate their pay-off from a fall-back payoff. The fall back position for each agent occurs when no bargain is reached. Let $V_0$ and $\pi_0$ represent the fall back level of regulator and the firm. The bargain satisfies:

$$\max_{x\,y} \Phi(x,q) \text{ where } \Phi(x,q)= (V(x,q)-V_0)^{\phi} (\pi(x,q)-\pi_0)^{1-\phi}. \qquad (41)$$

where $\phi\in[0,1]$ is a parameter that captures the bargaining power of the regulator. The first order condition, $\Phi_1 =0$, yields:

$$\phi\frac{xV_1}{V-V_0} = -(1-\phi)\frac{x\pi_1}{\pi-\pi_0} \qquad (42)$$

---

[9] There is no unique way to model the bargaining process. Many economists prefer to model it as a non-cooperative game (following Rubinstein 1982). However in this case it would be necessary to model the bargaining game. The cooperative approach avoids adding this additional (and to some extent arbitrary) detail. For the purposes of this paper the differences in approach is not material, as the outcome of bargaining always lies on the contract curve. Further, using the cooperative approach to bargaining may not be inappropriate as there is often a symbiotic relationship between the regulated and the regulator.

This first order conditions indicates that under the optimal bargain the uoq produced are increased to the point where the weighted percentage gain in consumer utility is just equal to the weight percentage loss of profit. Call (42) the optimal uoq curve. The first order condition, $\Phi_2 = 0$, yields:

$$\phi \frac{qV_2}{V - V_0} = -(1-\phi)\frac{q\pi_2}{\pi - \pi_0} \qquad (43)$$

This first order conditions indicates that under the optimal bargain qpi is increased to the point where the weighted percentage gain in consumer utility is just equal to the weight percentage loss of profit. Dividing (42) by (43):

$$\frac{V_1}{V_2} = \frac{\pi_1}{\pi_2} \qquad (44)$$

Equation (44) represents the contract curve, that is the points of tangency between the consumer's indifference curves and the iso-profit curves. Thus points along (44) represents set of efficient bargains. The optimal bargain simultaneously satisfies (42) and (44), where (42) can be interpreted as identifying the outcome of negotiations between the two parties over the set of efficient bargains.

## 3.2 The Contract Curve

The contract curve, (44), may be written as:

$$c_2 = p_2 x + \lambda V_2 = B_2 - (1-\lambda)V_2 \qquad (45)$$

as $V_1 = -p_1 x$ and where $\lambda(x,y) \equiv 1 - \left(\frac{p - c_1}{p}\right)\varepsilon_X$ is the ratio of marginal profit and marginal utility (with respect to x). Assuming bargaining results in price being greater than marginal cost, $0 \le \lambda \le 1$. It is reasonable to assume (and required by the second order conditions) that $\lambda_1 > 0$. $\lambda$ can be related, via (42), to the bargaining power of the regulator. If regulator has no bargaining power then the monopoly price is set and $\lambda = 0$. As regulator bargaining power increases x increases, and $\lambda$ also increases. If x is raised to the point where price equals marginal cost then $\lambda = 1$.

The second term of the RHS of (45) is $\lambda$ times the contribution of an increase in qpi to consumer utility. Thus the RHS of (45) can be interpreted as the weighted marginal benefit of qpi. The equation therefore states that the bargained qpi occurs at the point that the weighted marginal benefit of qpi equals the marginal cost of qpi.

As $0 \leq \lambda \leq 1$, (45) shows the weighted marginal benefit is positive if consumers have an autonomous taste for qpi ($B_2 > 0$). Thus:

> Proposition 13: When consumers have an autonomous taste for qpi the optimal bargain yields a level of qpi greater than the cost minimising level. However when consumers do not have an autonomous taste for qpi ($B_2 = V_2 = 0$) the optimal bargain yields the cost minimising qpi.

An implication of Proposition 13 is that when consumers have an autonomous taste for qpi then, as shown in figure 4, the contract curve lies above q*(x), the cost minimising qpi curve. When consumers have no autonomous taste for qpi the contract curve coincides with q*(x).

Comparison of (45) and (34) shows the monopolistic outcome ($\lambda = 0$) lies on the contract curve. Further comparison of (45) and (39) shows the efficient outcome ($\lambda = 1$) lies on the contract curve.

The slope of the contract curve therefore indicates how a movement from monopolistic outcome towards the efficient outcome affects qpi. The slope of the contract curve is given by differentiating (45):

$$\frac{dy}{dx} = \frac{p_2 + (1-\lambda)xp_{21} + \lambda_1 V_2 - c_{21}}{c_{22} - p_{22}x - \lambda V_{22} - \lambda_2 V_2} \qquad (46)$$

using $V_{21} = - xp_{12}$. Therefore the contract curve is positive provided:

$$p_2 + \lambda_1 V_2 + (1-\lambda)xp_{21} > c_{21} \qquad (47)$$

as $c_{22} - p_{22}x - \lambda V_{22} > \lambda_2 V_2 = \left(\frac{p_2 - c_{12}}{p_1 x} - \left(\frac{p - c_1}{(p_1 x)^2}\right) p_{12} x\right) V_2$ in order to satisfy the second order conditions. The LHS of (47) is the upward shift in the weighted marginal benefit curve. Observe that the sign of $\lambda_1 V_2 + (1-\lambda)xp_{21}$ is ambiguous, so that an increase in x need not shift the weighted marginal benefit curve upward. However if the marginal benefit curve does shift upward following and increase in x, the contract curve is upward sloping if $c_{12} \leq 0$.

It was shown above that $q^e > q^m$ if $V_2(x^m,q)>0$ and $c_{12}(x,q) \leq 0$. It might be thought that the contract curve is positively sloped if $V_2(x,q)>0$ and $c_{12}(x,q) \leq 0$ for all $x \in [x^m,x^e]$ and $q \in [q^m,q^e]$ . However figure 5 shows this is not necessarily the case. Figure 5 depicts the marginal cost of qpi curve, $c_2(x,q)$ and the weighted benefit, $p_2 x + \lambda V_2$. Consider an increase in the uoq produced from x to $x^+$. If $c_{12} \leq 0$, $c_2$ shifts to the right. However if $p_{12} < -(p_2 + \lambda_1 V_2)/[(1-\lambda)x] < 0$, the weighted benefit curve shifts downward. If this downward shift is sufficiently great (ie if $p_{12}$ is sufficiently negative), then the bargained qpi falls, and thus the contract curve is negatively sloped. However if $\lambda$ is sufficiently close to 1, the weighted benefit must shift upward. Thus, by (47), if $c_{12} \leq 0$ the contract curve is upward sloping at the point $(x^e,q^e)$.

It was shown above that $q^e < q^m$ if $V_2(x^m,q)<0$ and $p_2 - c_{12}(x,q) \leq 0$.These conditions are not sufficient to ensure the contract curve is negatively sloped. In particular if:

$$- (\lambda_1 V_2 + (1-\lambda) x p_{21}) < p_2 - c_{21} < 0 \qquad (48)$$

the contract curve is upward sloping. If $(\lambda_1 V_2 + (1-\lambda) x p_{21})>0$ the weighted marginal benefit curve shifts upward following an increase in x. However if $c_{12} \geq 0$ the slope of the contract curve is ambiguous for reasons analogous to those used in analysing figure 2 (with the benefit curve in figure 2 replaced by the weighted benefit curve).

Proposition 12 provides a condition when $q^e > q^m$ if consumers have an autonomous taste but no autonomous demand for qpi. Consideration of (48) yields:

Proposition 14: Under the conditions of Proposition 12, the contact curve is downward sloping for any region for which $c_{21} > \lambda_1 V_2 > 0$ and upward sloping for any region for which $c_{21} < \lambda_1 V_2$.

Thus if consumers have an autonomous taste, but no autonomous demand, for qpi the contract curve is upward sloping if cost is additively separable. However the contract curve need not be upward sloping if cost is multiplicatively separable.

The optimal bargain simultaneously satisfies (42), the optimal uoq produced curve, and (45), the contract curve. The contract curve ensures that all gains to trade are exhausted, while the optimal uoq produced curve identifies which one of the efficient bargains is actually negotiated. A contract curve, with $q^e > q^m$, is shown in figure 4. The contract curve is also shown to have a negatively sloped region. As noted above, these assumptions are not inconsistent with $V_2(x^m, q) > 0$ and $c_{12} < 0$. Additionally, an optimal uoq curve with assumed regulator bargaining power $\phi_1$, $\Phi_1(\phi_1) = 0$, is depicted. With regulator bargaining power $\phi_1$, the optimal outcome is $(x_1, q_1)$.

The impact of an increase in the bargaining power of the regulator can be analysed using figure 4. An increase in the bargaining power of the regulator, from $\phi_1$ to $\phi_2$, shifts the optimal uoq curve upward. As a result the optimal bargain moves to $(x_2, q_2)$. In the case depicted in figure 4, the increase in bargaining power of the regulator shifts the optimal uoq produced curve up over the negatively sloped region of the contract curve, hence qpi falls. Thus an increase in the power of the regulator, even under the condition $c_{12} < 0$, may see qpi fall. However it should be noted that such a movement improves both consumer and social welfare. Nonetheless, it is also clear from figure 4 that a sufficient increase in the bargaining power of the regulator, by moving x toward $x^e$, eventually causes qpi to rise toward $q^e$.

## 4. Conclusion

This paper offers some refinements to full information models of monopoly choice of qpi. It does so by re-specifying consumer preferences in terms of the presence or absence of both an autonomous taste for qpi and an autonomous demand for qpi. This reformulation suggests that consumer surplus may include an autonomous component, which has not been incorporated in previous work. The classic question, of how the monopoly qpi compares to it efficient qpi, is then considered. The approach adopted in this paper highlights, as best as seems possible, the separates roles of firm costs and consumer preferences in determining the relative size of the monopoly and efficient qpi. In particular if a consumer does not have an autonomous taste for qpi then the monopoly qpi and efficient qpi are those that are cost minimising for their respective number of uoq. In general the cost minimising qpi varies with the uoq produced. (For this reason, considerable attention is given to the specification of the cost function in section 1.2). Only if the cost minimising qpi is independent of uoq does a monopoly produce the efficient qpi.

If consumers have an autonomous taste for qpi, but no autonomous demand for qpi, then they necessarily have an autonomous component of consumer surplus. This possibility has been overlooked in the literature. However it is arguably realistic for many applications, particularly for natural monopolies. With these consumer preferences, the monopolist chooses the cost minimising qpi (given it produces the monopoly uoq). (Intuitively the monopolist's revenue is invariant to changes in qpi.) However, because consumers have an autonomous taste for qpi, they receive a benefit from qpi on top of that associated with their consumption of uoq. Therefore the efficient qpi is greater than the cost minimising level. Thus if the efficient cost minimising qpi is no less than the monopoly cost minimising qpi, the efficient qpi is greater than the monopoly qpi.

The assumption that consumers have an autonomous demand for qpi is more restrictive than the assumption adopted in the existing literature (ie that $P_2>0$). An implication of this assumption is that it both the monopoly and efficient qpi are greater than their respective cost minimising levels. (Under the usual assumptions in the literature that $P_2>0$ it is possible that qpi could be below the cost minimising levels.)

However there remains ambiguity as to the relative size of the monopoly and efficient qpi.

Nonetheless it was shown if marginal consumer surplus with respect to qpi is positive (at the monopoly uoq) and cost is additively separable the efficient qpi is greater than the monopoly qpi. Furthermore, if the cross partial derivative of the cost function, $c_{12}$, is sufficiently negative then the efficient qpi is also greater than the monopoly qpi. On the other hand if $c_{12}$ is sufficiently positive then the monopoly qpi is greater than the efficient qpi. Note, however, that efficient cost minimising qpi places a lower bound on possible values of the efficient qpi.

The analysis in this paper shows that ambiguity of relative size of the monopoly and efficient levels of quality arise because both the marginal consumer surplus with respect to quality, $V_2$, and $c_{12}$ have ambiguous sign. This contrasts with the formulation in existing literature that emphasises on the role of $P_{12}$, the change in the slope at each point along the inverse demand for noi curve as qpi changes, as a cause of the ambiguity. Focusing on $V_2$ is superior in theory because it allows for the possibly that consumer surplus can have an autonomous component (such as is the case when consumers have an autonomous taste but no autonomous demand for qpi.) This is shown by (22). Using $V_2$ is also likely to be superior in practice, as it only requires identifying the change in consumer surplus at the margin. This could be done using a variety of survey based mechanisms.

This paper also modelled regulated public utility. The regulator is assumed to be a consumer advocate, and bargains with the firm over quantity and quality. In this case the optimal bargain is shown to lie on the contract curve. The contract curve connects the monopoly outcome with the efficient outcome. However conditions which guarantee $q^e > q^m$ do not necessarily guarantee that the contract curve is upward sloping. For example if consumers have an autonomous taste for qpi the contract curve corresponds with the locus the cost minimising qpi levels. However, the cost minimising qpi need not be monotonically increasing or decreasing as uoq increases.

It is shown that if consumers do have a autonomous taste for qpi, then the optimal bargain specifies a qpi greater than the cost minimising qpi. The slope of the contract curve is ambiguous even when $V_2$ and $c_{12}$ can be signed. However if consumers do not have an autonomous demand for qpi and $c_{12}<0$, then the contract curve is upward sloping.

Identifying the slope of the contract curve is important as it indicates how an increase in the bargaining power of the regulator affects qpi. In particular it indicates the extent to which a move toward efficiency is expressed as an increase in qpi or as a decrease in price of quality.

The reformulation presented in this paper does not remove the ambiguity (concerning the monopoly and efficient qpi) that led Spence to describe his results as "somewhat discouraging" (p.428). However it is the contention of this paper that too much pessimism is unwarranted. In many instances it suffices to model consumers as having either no autonomous demand for qpi or no autonomous taste for qpi. In these cases, knowledge of the cost function allows identification of the relative magnitudes of the efficient and monopoly qpi levels.

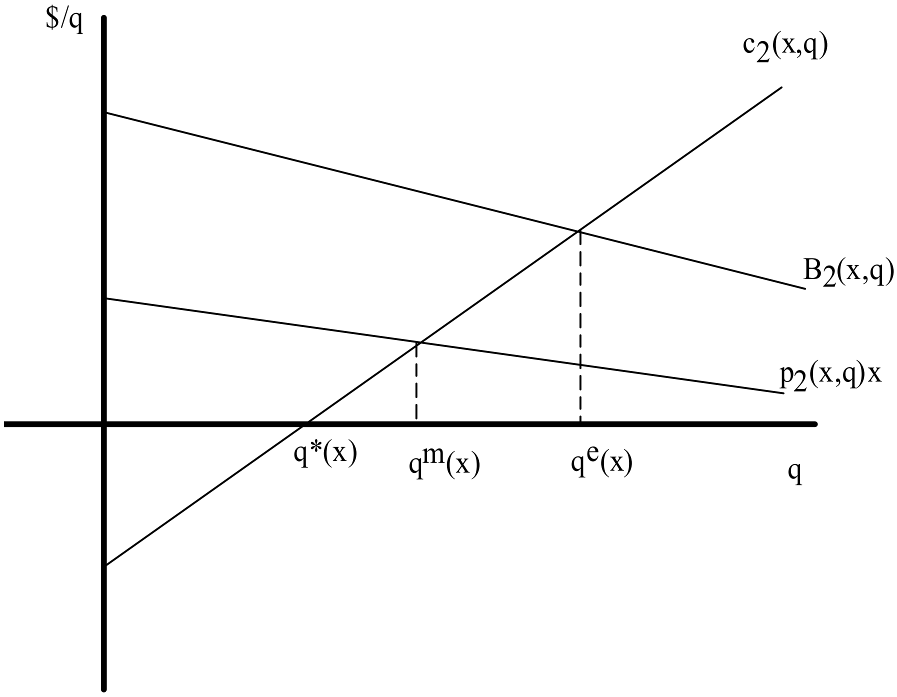


Figure 1: Choice of qpi given
uoq when $V_2>0$.

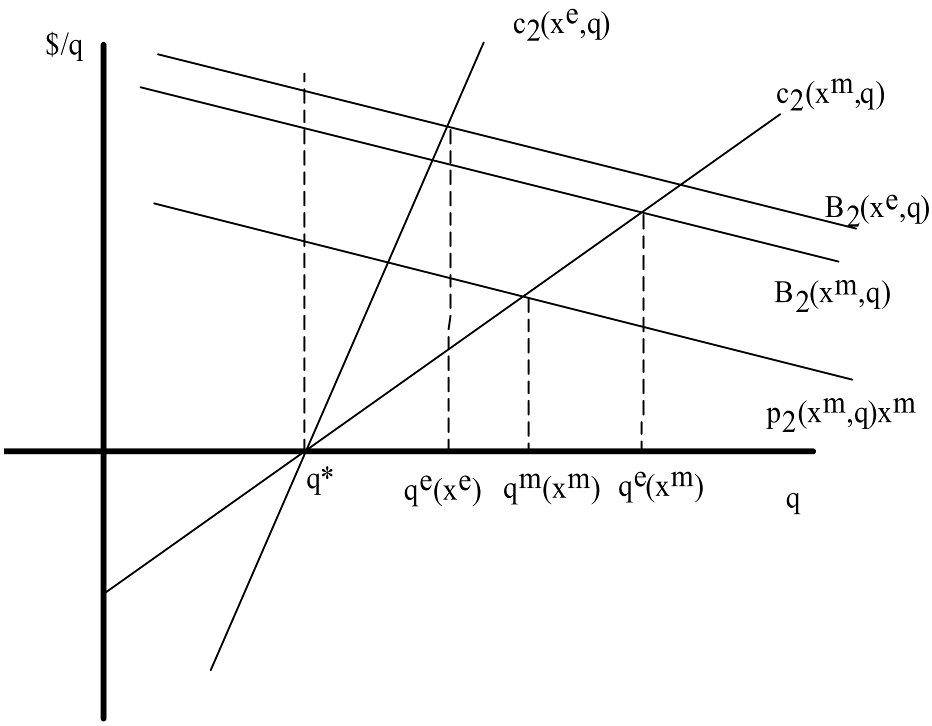


Figure 2: Monopoly vs. Efficient quality
with multiplicatively separable cost, $V_2>0$, and large $\chi$.

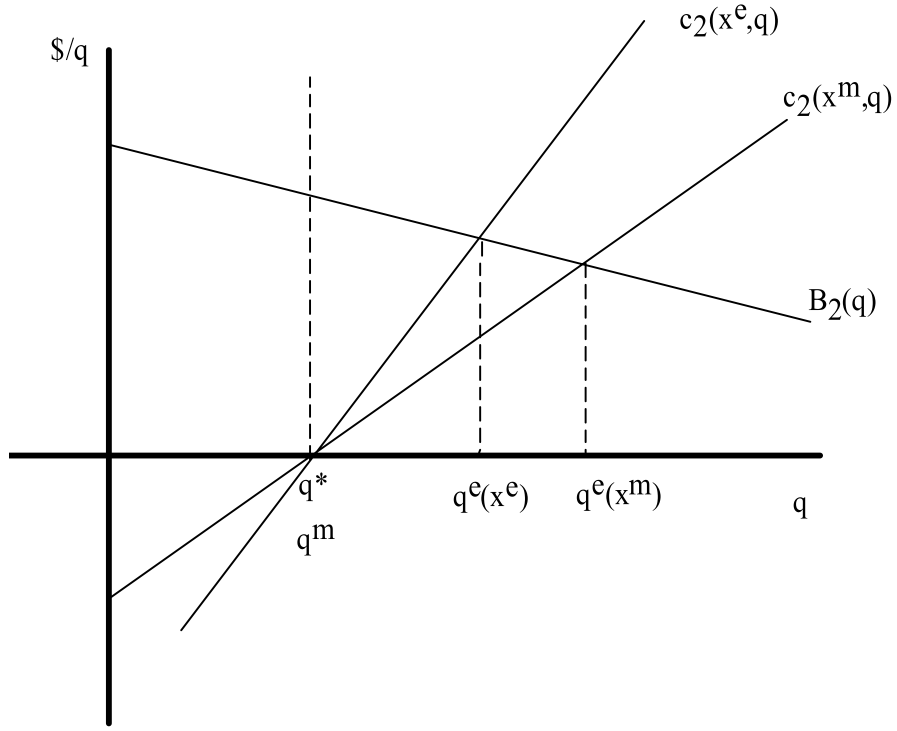


Figure 3: Monopoly vs. Efficient qpi
when there is no autonomous demand for qpi.

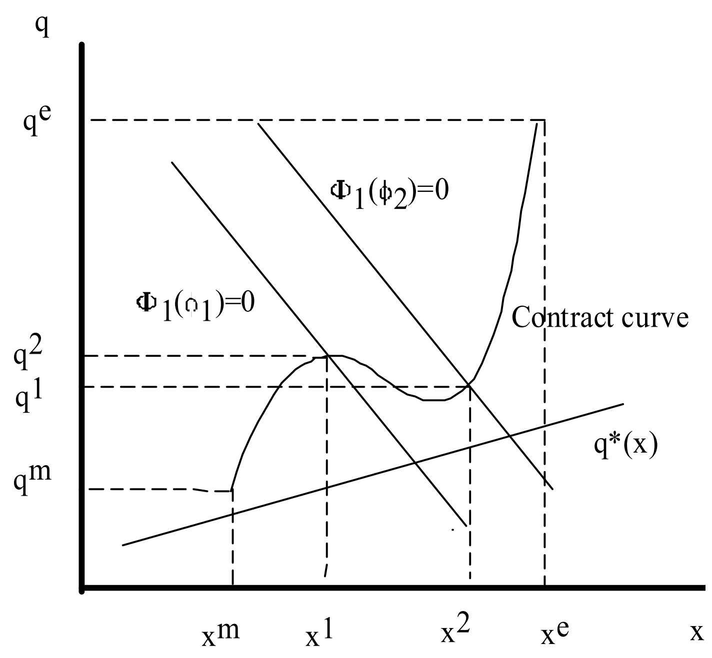


Figure 4: An increase in the bargaining power
of the regulator, from $\phi_1$ to $\phi_2$, when $c_{12}<0$.

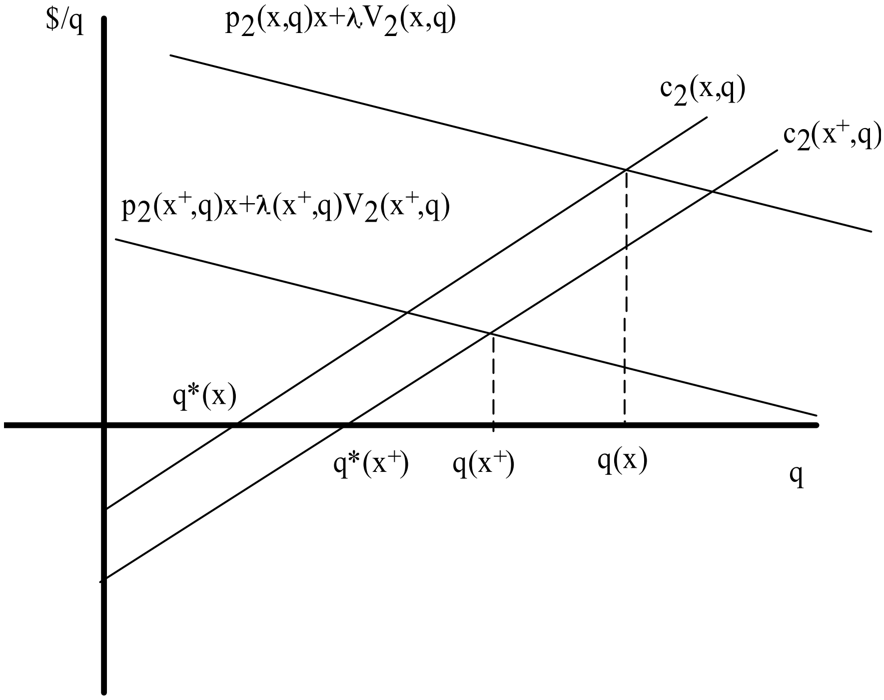


Figure 5: Change in qpi along the contract curve
when $p_{12}\leq 0$ and $c_{12}\leq 0$.

## References


Dorfman, R. and Stiener P. O., (1954), "Optimal advertising and optimal product quality", *American Economic Review*, 44, pp.826-36.

Eichberger, J., (1993), *Game Theory for Economists*, San Diego: Academic Press.

Kihlstrom, R. E. and Levhari, D., (1977), "Quality, Regulation and Efficiency", *Kyklos*, V30, pp.214-34.

Mussa, M. and Rosen, S., (1978) "Monopoly and Product Quality", *Journal of Economic Theory*, 301-17.

Rubinstein, A. (1982), "Perfect Equilibrium in a Bargaining Model", *Econometrica*, 50, pp:97-109.

Schmalensee, R., (1979), "Market Structure, Durability and Quality: A Selective Survey" *Economic Inquiry*, V. XVII, pp. 177-96.

Shaked, A. and Sutton, J. (1983), "Natural Oligopolies", *Econometrica*, 51, pp. 1469-84.

Sheshinski, E., (1976), "Price, Quality and Quality Regulation in Monopoly Situations", *Economica*, 43, pp.127-37

Spence, A.M. (1975) "Monopoly, quality and regulation", *Bell Journal of Economics*, pp. 417-429.

Swan, P., (1970), "The durability of consumption goods", *American Economic Review*, pp.884-94.

Tirole, J. (1988), *The Theory of Industrial Organization*, MIT Press, Cambridge, Massachusetts.

## Economics Discussion Papers



Copies of the above mentioned papers and a list of previous years' papers are available on request from the Discussion Paper Coordinator, School of Economics, University of Tasmania, Private Bag 85, Hobart, Tasmania 7001, Australia. Alternatively they can be downloaded from our home site at http://www.utas.edu.au/economics