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VISUALIZING THE GAINS FROM TRADE, MID-1870s T0 1962

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*The text is mine, though it bears in subtle ways the marks of Roy Weintraub's influence, spread over many years. I am grateful to him in this instance for allowing me to try and engage him in the topic, and for lending his ear and keen mind in discussions concerning it. He read a large pile of things I gave him and also plied me with references I would not have encountered on my own. His quietly-made suggestions have repeatedly challenged and frequently enlarged my thinking. Torsten Schmidt introduced me to Edgeworth's review of Auspitz and Lieben; Robert Leonard supplied helpful critical comments on a late draft; and audiences at Duke University and UQAM, listened to and commented on successive versions. My thanks to them all.

Introduction

Marshall translated Mill's discussion of the distribution of the gains from trade into visual form, and visualization remained a primary vehicle for the analysis and presentation of the gains and their allocation for the better part of a century. Why was that so? It is unlikely that it was due to some peculiarly visual core in the idea. Marshall's "net benefit from trade" is something quite different from its successor, the analysis of the proposition that some trade is better than no trade that we owe principally to Viner and Samuelson. Indeed, a striking thing about the period mid-1870s to 1962 is that the tools used to analyze the gains from trade and the contexts from which they arose altered sharply even as visualization remained a preferred means. This, plus the fact that the preference itself was selective, tells us that the persistence of visual means almost certainly owed more to audience capabilities than to any stable and common visual core in the gains concepts themselves.

The notion of visual core or essence does have a history, going back to Plato's ideal forms. But as recent neurological research, and reinterpreted older evidence, clearly suggests, our capability for recognizing and conceptualizing forms is based on forms previously seen and stored in the brain. A great part of the visual potential a normal person possesses is there at birth, though its early nourishment is also essential. Deprivation in the first days and months, or later lesions, can destroy the functioning of cells that in a normal brain are "the building blocks for the perception of forms" (Zeki 1999, 93-4). In the absence of prior exposure to forms, proper early nurturing of the ability to grasp them, and healthy brain cells of the right sorts, talk of ideal forms literally makes no sense.

This conclusion affects negatively the concept of *ideal* forms but also the musings of various modern artists concerning universal *natural* forms, which some purported to find in linear abstractions such as the vertical and horizontal. It turns out that there is a predominance in the visual cortex of cells that respond selectively to lines of specific orientation, but this too says more about how neurobiology shapes our ability to construct visual meaning out of the bits of visual stimuli we receive, than it does about the validity of these artists' reflections (Zeki op. cit., 109-13).

Current neurological thinking holds that, after due nourishment during the critical early period, connections are also stabilized between eye and brain (Zeki op. cit., 94). Yet just as facility in language is not rendered uniform by a certain universality in grammatical structures, so this stabilization does not rule out differences in visual aptitude between individuals or negate the role of education in refining our ability to interpret visual signals, to give them specific content in this context or that. The professor of neurobiology at University College, London, Semir Zeki, whom I have been citing, is deeply interested in the neurology of art, and he finds in later Cubism[1] and in the paintings of Vermeer an implicit appeal to such differences; there is in their works, he says, a "cultivated…ambiguity." By ambiguity here Zeki means neurological open-endedness (26-9). The viewer is asked, even required, to become "imaginatively involved" in these paintings, as is the case also with the films of Antonioni (26).

The equivalent of this insight applied to visualizations of the gains from trade would be a simple recognition that the viewer is an active participant in appropriating, filtering, and re-composing the stimuli received from what we have too often unthinkingly regarded as univocal, because "scientific," diagrams. The implication is not, however,

[1]Picasso's *Man with a Violin*, in the Philadelphia Museum of Art, is chosen by Zeki as an example.

that diagrams can be given different "meanings." It is, rather, that particular visualizations are not necessarily complete, do not quite work, seem to have something about them that demands further exploration, requires experimenting with. And this has two components. It can be understood from the maker's side; we see in Samuelson's visualizations of the gains from trade something of experimentation, moves that were not truly convincing, and an absence of a clear sense on his part of a single path to successful visualization. But often there is work for the viewer to do as well. Whether a particular visualization is compelling is not just for the maker to determine. Visualizations, offering as they do a range of stimuli, will be appropriated and put together differently in the brains of different viewers, reflecting individual and group capabilities, which in turn depend on prior exposure, memory, context and imposed selective emphasis.

If this is true, we have now acquired two reference points. One, due to my earlier borrowings from Zeki, is that diagrams (of whatever sort) are not for everybody: [2] exposure, and apprenticeship in appropriation and interpretation, make all the difference. Second, since diagrams are abstractions and are also made up of various components, even among well-equipped viewers there may be differences in the degree to which hidden elements are "visible," and in the force with which a particular visualization stimulates, informs and persuades.

In my history of visualizations of the gains from trade I will try to allow for these inevitabilities and possibilities. Within the time span mid-1870s to 1962 there appeared a surprising range of such visualizations, many of them noted by Viner in his masterly critical survey, *Studies in the Theory of International Trade* (1937). However, I do not

[2] They are, however, and very definitely for some. During the discussion following a recent talk at Duke University by Robert Lucas on the Chicago of the 1960s, it was recalled that a student caricature of Harry Johnson portrayed him as an octopus, with a pencil in every tentacle.

seek completeness, and my title is thus misleadingly broad; nor do I wish to create a connected sequence that might leave an impression of unbroken progress at a logical level. Instead, and to illustrate especially the possibilities just outlined, I have selected just two visualizations, one by Marshall, and dating from some of his earliest explorations of value in domestic and international contexts – hence the mid-1870s start to our period – and one, a process in fact, involving Samuelson, ending with his second attempt to demonstrate that some trade is better than no trade, in 1962. The differences between these two sorts of visualization are stark enough to make the point that the component ingredients and problem contexts out of which visual understandings are shaped matter in ways that may seem obvious once stated, yet are also powerful enough to eclipse the view that there is some universal, natural way to show, much less "see," a particular phenomenon such as the gains from trade.

***Real*-ising: Marshall on diagrams**

One should be wary of attributing purpose; nevertheless, Marshall lends himself to characterization as one who hankered after creating analysis that would be helpful to those having to shape policy. Diagrams seem to have appealed to him as useful for this purpose, and nowhere more so than in the area of trade. He wrote in *Money, Credit and Commerce* (1923) that it is most effective to represent by diagrams "the exclusive trade between two countries;" for "details [such as those involved in tracing the possible impacts of trade taxes] can be handled more thoroughly and securely, and at the same time really more easily, with the aid of diagrams" (161, 187).

The alternative to curves was mathematics. In his *Essay on Value* (early 1870s?) Marshall acknowledged that diagrams do not possess either the "generality" or "minuteness" of conclusions reached by mathematical methods (Whitaker 1975, I, 156). Nor do diagrams yield proof; they are just "illustrations…merely pictures corresponding to the main conditions of certain real problems. They obtain clearness of outline, by leaving out of account many considerations which vary from one practical problem to another" (*Principles*, 9th, variorum, ed., 155, n.2). At the same time – this from his *Pure Theory of Foreign Trade*…[and] *Domestic Values* – "[d]iagrams present simultaneously to the eye the chief forces which are at work, laid out, as it were, in a map; and thereby suggest results to which attention has not been directed by the use of the methods of mathematical analysis" (1930 [1879], 5). In this vein, and although Marshall was reticent about applying curves to "market"-period analysis, where things are either "absolutely abstract" or "terribly concrete," he granted in a letter to Edgeworth that even there they have the great value of enabling one to keep in view the "concrete and…ever-varying (though individually vital) side issues" (quoted by Whitaker op. cit., I, 123).

Diagrams for Marshall, it seems, contributed a kind of inductive analysis that was critical to the research enterprise. By illustrating case after case using differently shaped and sloped demand and supply curves, for example, one could trace various possibilities, and after making an "enormous number" of these "maps" there might emerge "approximate generalizations" (Whitaker op. cit., I, 156). Once such inductive work has been done the "natural superiority" of mathematics would assert itself: the pattern in particular results merely "shadowed forth" in diagrams would then come into plain view, "represented by mathematical curves" (ibid.). Yet, tedious, imprecise and uncertain

though the inductive phase must be, to go straight to the mathematics would be wrong. Without the effort of attempting to represent case after case of real possible circumstances and the forces that shape them, mathematics would possess no practical value; mathematicians would find themselves merely "analyzing the results of arbitrary hypotheses" (ibid.). This inductive aspect, Marshall wrote to Jevons (though not in those words, and no doubt with more than a dash of false modesty), would be his "mite," his contribution "towards that work of '*real*'-ising the results of abstract quantitative reasoning in Economics of which I recognize in you the chief author" (quoted in Whitaker, op. cit., I, 85, emphasis added).

In terms of trade policy, and as hinted at already, one of the issues that engaged Marshall was the impact of import duties. Could a tariff be imposed that would be borne wholly by the foreign trading partner? This had long been an analytical question, but was also an aspect of actual tariff reform debates, such as the one which erupted in the early years of the new century. In his privately circulated *Pure Theory of Foreign Trade*…[and] *Domestic Values* (op. cit., 269-71) Marshall used offer curves to address the question. A tariff would make England's offer curve, shown as rising with increasing slope from the origin towards the north east, steeper at every point. Only if the partner's – here Germany's – offer curve, rising with decreasing slope from the origin north-westerly, was completely flat in the range affected by the tariff, would it pay all the trade tax. For then Germany would get fewer E-bales (Marshall's summary indicator of England's exports), but without giving up any fewer G-bales. With slightly different assumed shapes in the relevant range, Germany might bear some or none at all of the tax burden. The example bears out all the concerns expressed above, among them the

importance of spelling out the chief possibilities and the circumstances which might give rise to them, before any worthwhile policy advice could be offered.

Measuring "net benefit from trade"

Whether there are gains from trade, and how large, of course are considerations prior to the issue of how such gains might be divided, including whether a trading partner might be made to bear the whole burden of a tariff. There too Marshall's preferred visual tool was the offer curve, which is a locus of supply-*and*-demand offers at specified terms seen from the point of view of one trading partner. The intersection of offer curves, one for each trading partner, marks a point of "general" (two-party) trade equilibrium.

Had Marshall wished to identify the gains from trade with labor-cost savings, he might have made do with a Ricardo-like visualization in terms of a straight-line production possibilities frontier and an enlarged consumption possibilities frontier due to the improved terms of exchange available under free trade. Comparing the new with the original availabilities of goods at the pre-trade terms of exchange indicates a goods-equivalent or, under given technology, a labor-equivalent of the expanded income enjoyed through trade (as in Findlay 1970, 110). Instead Marshall stuck with Mill's offer curve construct. Why? In part, surely, to ensure a role for demand as well as supply; but also, I suggest, because an offer curve describes the various trades a partner is willing to make, including of course their terms, which will vary at every point along a continuous curve. Marshall could thus bring to bear a notion like consumer's surplus, where the surplus arises as a series of differences between willingness to pay on the way to equilibrium and the final agreed terms at which trade will actually take place.

Viewed in this way, at every point on an offer curve prior to trade equilibrium there is a positive, though steadily decreasing difference between willingness to pay and what in fact must be paid in equilibrium. By analogy with consumer's surplus, the total gain might be thought of as the sum of the differences between willingness to pay and required payment, in the trade case each expressed relative to the total volume of imports at equilibrium.

Auspitz and Lieben independently arrived at this insight, using a construct that employs money prices and is thus a bit closer to the demand curve, while making Marshall's notion of net benefit from trade relatively easy to follow. Here I adopt Edgeworth's exposition in an 1889 review in *Nature*, of *Untersuchungen über die Theorie des Preises* (Edgeworth 1889, 242-43). In the diagram below the x-axis measures amounts of a commodity, the y-axis amounts of money. Then, QR stands for the amount of money which a consumer would be willing to give in exchange for OQ of the good. It is assumed that the consumer is indifferent between buying on these terms or doing without. Call the locus of such points ON the utility curve. ON′ beneath ON is the demand curve, giving the amounts our individual is willing to buy at various prices, each represented by the Tan of the angle R′OQ. Supposing the marginal utility of money to be constant, the individual gains a sequence of lengths RR′ equal to the difference in Tangents (e.g., of ROQ versus R′OQ), or, more strictly, of lengths such as RR′ relative in each instance to the amount of the good involved at that point.

Figure 1, Auspitz and Lieben's visualization of utility, demand and net gain

[figure here]

Source: Edgeworth, review of *Untersuchungen über die Theorie des Preises*, *Nature*, July 11, 1889, 242.

Marshall's own visualization of the net benefit, I have said, is more difficult to appropriate than the one just given. Possibly this stems from his not keeping distinct the utility and demand curves, as is done in fig. 1, but making the offer curve do double duty, as both. However that may be – viewers will hold different opinions on ease or difficulty – Marshall definitely made his diagram cumbersome by adding a visualization of the point by point benefit, first as a sequence of vertical distances, then as a single composite ratio (gain divided by total imports). The final diagram thus seems to incorporate in one something of both the inductive research phase and the mathematical curves phase in which those results are summed up.

Marshall first published his visualization in *Money, Credit and Commerce* (1923, 338-40), though it was developed in the 1870s (Whitaker op. cit., I, 280-81). Viner (1937, 570-71) effected slight changes that make Marshall's approach a little easier (for me) to follow, and I reproduce his variant here. The accompanying exposition can be paraphrased as follows. OG is Germany's offer curve, and in free trade equilibrium Germany exports OH (=AB) G-bales for OB E-bales. OR is the tangent to OG at O, and at this point Germany gains RA over the equilibrium terms of trade, which are given by the Tan of the angle AOB. At each potential exchange point on the way to equilibrium the gain will be less, finally disappearing altogether. Thus at P, it will be pA (=P′M′), or

a gain at the *rate* of pA (=P′M′) on OB E-bales, the final amount of imports agreed to. The gain at P is then the OBth part of P′M′ or P′M′/ OB. If P, beginning at O, is made to move along OG, P′ will start from U and trace out the lengths of the gain at each point by the curve UP′A, ending at A. Draw VW parallel to OX, such that the rectangle VHAW is equal to the area UHA. Then Germany's net benefit from trade with England will be the OBth part of VHAW, which is VH.

Figure 2, Viner's variant of Marshall's net benefit of trade diagram

[Viner's figure here]

Source: Jacob Viner, *Studies in the Theory of International Trade*, New York: Harper and Brothers, 1937, Chart XVI, 571.

Even as he gave it as his opinion that diagrams are the best way to represent trade between two countries, Marshall also allowed that "diagrams are not effectively assimilated by all readers" (1923, 161). In the case of his own net benefit diagram there was a particular reason why assimilation might have been difficult. For it employed a close analogue of a procedure from seventeenth-century mathematical practice familiarity with which, even among Cambridge graduates, must have been in increasingly short supply by 1923.[3] But even for those versed in geometry Marshall's procedure was multi-layered and required sustained concentration.

In Marshall's student days problems in the mathematical Tripos were drawn mostly from mechanics, though far enough removed from any real situation to make them

[3] Cambridge had seen successive reductions since the mid-nineteenth century in the mathematical requirements for general students at the University; for details see Groenewegen (1995), 80-94, and Tribe (2000).

tractable. Importantly from our present perspective, the solution methods asked for involved geometrical constructions. For example, selected propositions of Newton were to be addressed and, as the 1849 Report of the Examination Board put it, also "proved in Newton's manner." To students of the *Principia* this would have been clear, though what was required was also spelled out in the Report: conic sections, though "treated geometrically;" "the elementary parts of statics and dynamics," again "treated without the differential calculus;" "the elementary parts of Hydrostatics," ditto; and so on (Weintraub 2002, ch.1, passim.).

Historian of mathematics Henk Bos has written of the 17th century that very many even among prominent mathematicians could only recognize and understand mathematical curves after being shown a geometrical derivation of them. He illustrates with the example of Leibniz, who literally "constructed" an exponential equation for a perplexed Christiaan Huygens, who had found the equation itself when presented very obscure, implying as it did a "sort of transcendental lines, in which the unknowns enter the exponent" (Bos 1993, 23). Huygens begged to be shown "the form of the curve by [your] marking points on it by whatever method" (24). He grasped the equation only when he had viewed Leibniz's construction of points on a curve that he called the *Logarithmica*.

Bos offers another illustration of such a construction procedure, this one due to Jakob Bernoulli who, in 1694, devised a solution curve for the differential equation for an elastic beam bent under tension and to which he applied Hooke's law, that extension is proportional to force (Bos op. cit., 31-2). The solution in this instance involved showing how an area under a known curve at any distance along the base can be translated into a

rectangle, whose height is a point on the solution curve. This was known as "construction by quadrature."

These examples illustrate well the importance of prior exposure to our being able to "see." Here, moreover, in an intriguing inversion of the order often presupposed among economists, visual exposure was prerequisite to mathematical seeing. Note that the procedure last described is not quite the one Marshall employed: he knew both the curvature of the offer curve and, empirically, that of UP′A. So he did not need quadrature to construct the curve UP′A. Instead he used rectangulation to generate a precise composite measure of the net benefit: the vertical side of the rectangle equal in area to UHA (fig. 2). This aspect of the construction is not difficult to follow, but it requires patience, and the diagram itself becomes cumbrous through this element's being piled onto the already heavily-laden offer curve construct.

Marshall himself seemed to be aware that the exactitude promised in his summation of the gains, and which had occasioned the rectangulation, was out of his reach. There was an unbridgeable gap between, on the one hand, his exploitation of a psychological motivation of the measure itself and the particular want (and underlying good or goods) this implied, and, on the other, his use of aggregates for all exports and imports. Marshall noted to Cunynghame (quoted in Whitaker op. cit., I, 261) that this aggregation posed a problem. We also know that he found his own trade curves "very troublesome" for the purpose of showing benefit as a special kind of consumer's surplus (280). Viner made much of this troublesomeness in a criticism articulated in his 1937 survey of the gains literature.

Chicagoan scepticism: Viner's reservation about Marshall's measure, and his rejection of opportunity cost

Viner claimed that Marshall employed an interpretation of the offer curve which treated every offer as independent of the others. This, however, is invalid. For an offer curve is not a demand curve for a single commodity, and it is stretching credibility to assume that along it the marginal utility of money will remain constant. Put differently, since payment along an offer curve is in terms of an aggregate of exportables, it is unlikely that the marginal utility of remaining amounts of the aggregated "exportable" (G-bales in Germany's case) will be unaffected by prior sales and hence remaining amounts of G-bales. More probably, as the remaining number of G-bales declines, their marginal utility will rise, and Germany's willingness to pay will also decline – below what it would be had prior sales all been at the final equilibrium terms of trade. Marshall, however, implicitly considered each offer on the trade curve to make no difference to marginal utility, as a result of which he had most likely overstated the net benefit from trade (Viner 1937, 571-72). Marshall's supposition would become the "independent utilities" assumption in Paul Samuelson's criticisms of utility thinking.

This was just one of several conceptual objections raised by Viner. For example, he also complained that, whereas offer curves are "general equilibrium" in their implications because they describe supply-cum-demand offers, in Marshall's usage this general equilibrium character is treated casually. Not only were demands and supplies themselves aggregates, but there was no way in the Marshallian visualization to display the readjustments to domestic production implied by every change in relative prices (as along an offer curve). In this sense, "Marshall allowed the operations of the internal

machinery to remain unseen" (ibid., 585, n.3) and thereby, we might add, abandoned much of the advantage in a good diagram, that it can "present simultaneously to the eye the chief forces which are at work."

Yet Marshall was not Viner's main target. He objected on similar grounds – that the behavior of too many affected variables lay hidden in the visual construct used – to the newer opportunity cost approach to thinking about the gains from trade. In the early 'thirties this approach had become part of a striking visualization, familiar to generations of later students and effected through the marriage of Haberler's (convex from above) transformation curve (1930) with (concave) consumers' indifference curves. Viner himself may have been the first to put these two elements together in a single diagram, in a lecture he delivered at the London School of Economics, in January 1931. That diagram is shown here as figure 3 (Viner op. cit., 521 and note).

Figure 3, Viner's gains from trade chart prepared for and presented at the London School of Economics, January 1931

[Viner's Chart XI here]

Source: Viner, *Studies in the Theory of International Trade*, 1937, 521.

Ironically, since it became the basis of all later visualizations in our period, Viner invented and used this diagram only in order to make plain certain limitations of opportunity cost thinking, including modern utility analysis. The point K in Viner's figure is a point of equilibrium, as he explained, since there "the alternative costs and relative value of the two commodities would correspond." But this is a situation of

equilibrium prior to trade. If trade is opened up a new, with-trade equilibrium will be established, at production position G combined with consumption position H. H, being on a higher indifference curve, it indicates "a greater total utility" than before trade – or so the opportunity cost theorists would maintain.

Having stated this conclusion Viner immediately set about undermining it. First, he pointed out, a true whole-country indifference curve must refer to "a single valuing individual" – hence a one-person country or a land where all individuals are the same. If a whole community of distinct individuals is being supposed then a curve like MM′ must refer to equal-value combinations of the two goods when income distribution is such as to be consistent with the actual amounts of each produced in the absence of trade. But if the combination of goods produced alters, as it will under trade, there would be a different family of such curves. Some of these might intersect the set represented by MM′, due to distributional changes, an impossibility with "genuine" community indifference curves. The same applies to the supposedly higher-utility curve NN′, which therefore cannot "necessarily" represent greater utility or "real income". The opportunity-cost approach thus suffers on the income side the same difficulty as the real-cost approach on the cost side: neither contains information as to the proper weights to be applied whenever distributional issues arise (Viner op. cit., 523). On top of this, Viner maintained that the opportunity cost approach avoided supply-side problems, such as the effect of preference for leisure on labor supply, the allocation of factors across goods, and variable factor supplies, only by ignoring them (523-24).

But Viner was not only negative. He also managed to recast the whole issue of gains from trade by suggesting that, although one cannot be certain that trade is welfare-

increasing, certainly not for any particular individual, there will be more goods available with trade, hence there is the potential for redistributions to be effected such that, in principle, winners can buy off losers from the opening up of trade (Viner op. cit., 533-34).

A Samuelsonian basket of problems

This has been a slow-moving and possibly tedious stretch of exposition, made necessary by the distance in time back to Marshall's visual construct and the particular procedures he employed, with their even older roots. Viner's criticisms also make for slow going.[4] In turning now to Samuelson we can be quicker. We can also step back from what was threatening to become a narrative of the alternative conceptual merits of visualizations of the gains from trade, which was not my intention. Samuelson interrupts that flow in an interesting way.

Not only did he set off in an unexpected direction by dismissing utility theory as largely redundant, but he felt it necessary to supply micro-foundations for an already-existing visual tool in trade analysis, the production indifference curve, which had been drawn and adopted without its basis in efficient factor allocation having been openly examined. There were several other innovations on the conceptual path Samuelson cleared for himself. The whole process lasted many years, with a long gap between a promising early statement of the gains problem, in 1939, and 1962 when he suddenly

[4] Viner seems to have been quite at home with Marshall's diagrams; however, there is an interpretative barrier of sorts between us and Viner. His critical remarks are informed by a strong preference for partial equilibrium analysis. Confusingly, he used this to criticize equally Haberler and Marshall himself, as he thought necessary. More importantly, we have constantly to remind ourselves that Viner's Chart XI, which seemed so very modern to students of international economics of my generation, in his hands was a foil, designed to train a critical gaze on the new "general equilibrium" thinking embodied in the opportunity cost approach.

emerged with a stunning visual clarification (shown below as fig. 4). Why the gap? What altered between 1939 and 1962 that allowed a fresh "take" on the question? Why was his 1962 presentation wholly visual, whereas visualization had been only an occasional element in his progress to that point?

The answers to these questions are very instructive. Samuelson's 1962 restatement and summing up was a piece of visual high abstraction; but because we can trace the component, contributing bits of analysis that occurred along the way, his personal gains-from-trade odyssey reveals a construction process that seems to parallel what Zeki and others interested in the neurology of vision insist is typical (Zeki, op. cit., esp. ch.6, and ch.7, 71; also Hoffman 1998). They speak of functional visual specialization in different parts of the brain and of the involvement of networks formed by stimuli not only received at specific places but also transmitted to other parts. Out of these networks coherent visual images are formed, though how this occurs, and what might affect the speed of the process, is still little understood. They also stress that seeing and understanding cannot be separated, as many older thinkers would have it. There is no one part of the brain that pulls all the stimuli together, makes "sense" of them.

Early on in his career Samuelson enunciated a series of precise questions, equally careful answers to which he considered essential to clarifying what could be claimed for the gains from trade. Although these forays resulted eventually in his stunning visualization of 1962, how they would contribute to that particular bit of imaging could not have been anticipated when they were simply items on his youthful theoretical agenda. It should be stressed too, as I noted earlier, that visualization was not an active part of many of these component inquiries. Yet as the manner of their contributing

became gradually clearer to Samuelson himself, he was able to reach a point where seeing and understanding the gains from trade issue occurred together. By the end of the process he knew "the natural [visual] tool for th[e] purpose" (1962, 823), one that enabled everything important to be "specified at a glance" (827).[5]

It is possible to make a separate list of the questions Samuelson set himself to address during the extraordinarily productive years 1938-41, but these are so closely tied to his uncertain progress towards what in the end he deemed a satisfactory analysis of the gains from trade issue that I opt for considering both together. I will consider three early articles plus the one of 1962 on the gains from trade. Some more tedious verbal description is unavoidable in connection with the earliest of these papers, but the pace will quicken.

Viner, recall, had raised the possibility that society's utility (or indifference) curves might intersect if production (and distribution) alters when trade is introduced. For if uncontrolled (market-induced) reallocations are all that is available, and the wellbeing of those hurt by the move to trade dominates (because of the accident of preferences, or perhaps because this group has political power), then a with-trade equilibrium characterized by production at G and consumption at H (fig. 3) implies nothing about total utility. H's apparent superiority can be undermined by distributional considerations.

In February 1938, in an article in *Economica* titled "A Note on the Pure Theory of Consumer's Behaviour," a young but very self-confident Samuelson asserted that meaningful (refutable) propositions about consumer behavior (demand) could be reached without reference to utility, solely on the basis of observed data – total income, and

[5] I do not read "natural" here as having the same sense as I eschewed earlier. Samuelson's use of the word is quite specific, without wider implications.

relative prices – plus a single postulate: "the [idealized] individual must always behave consistently" (1938a, 62, 65). Consistency was defined in the following sense. Let a consumer buy a certain batch of goods at given income and prices. Then let the initial prices be combined with a second batch of goods. If this second batch costs the same or less than the first, both measured at the initial prices, then the second batch could have been bought initially, but was not. That it was not reveals that it was deemed inferior. It would be inconsistent for the same consumer to reverse him/herself and choose the second batch.

By contrast with this clear piece of deductive reasoning, Samuelson judged that we know nothing with certainty about "indifference directions" (slopes) or the way ratios of marginal utilities behave. The recent re-working of consumer theory by Hicks and Allen, postulating increasing marginal rates of substitution to account for concave indifference curves, thus seemed to him at best ambiguous and at worst an artificial convention that added no new information (1938a, 62).

The consistency postulate, often re-expressed (though with reservations) in index number terms, became a key element in Samuelson's later attempts to approach the gains from trade issue. The postulate would be applied to the gains question using original batches of goods and new prices, rather than new batches at original prices.

In an article in the June 1938 *American Economic Review*, "Welfare Economics and International Trade," Samuelson added a second key element. The argument to be made here was that, in a case where trade is considered to be barter between two individuals, "under free trade both parties are better off than under no trade at all" (Samuelson 1938b, 265). This was a strong normative result to envisage in the face of the unsettled

questions concerning utility and distribution that had so bothered Viner. How could it be successfully pursued?

Samuelson skirted the worst of the distributional problems by reasoning as if countries could be treated as individuals; and he avoided the real cost-opportunity cost disagreements between his Chicago teacher (Viner) and his Harvard professor (Haberler), by insisting that the proper way to get at whether one situation is superior to another is by introducing the first-order conditions of an optimum. That is to say, if specifying the conditions of general equilibrium is essential to setting up the question in a meaningful way, the real cost-opportunity cost debate loses its point. The conditions for an optimum constitute the second key element alluded to above.

As an aside, Samuelson noted that he would not attempt to measure any gains from trade. Marshall's measure was undermined by his having assumed "independent utilities" (recall Viner's criticism of Marshall's interpretation of offer curves) and constant marginal utility of money (1938b, 263, n.3).[6] Moreover, he (Samuelson) would invoke only an ordinal scale of preference, and define this scale broadly so as to include preferences involving the various kinds and amounts of productive services that an individual might choose to supply (262), here too reflecting Viner's concerns. The scale being ordinal, moreover, he would not try to use index number analysis in any sense other than to indicate direction of change – more preferred versus less preferred (263 and n.3).

But what then was the core of Samuelson's modified approach? It was to pose the question of whether there are gains from trade as a general equilibrium problem; more precisely, as one to be judged from the perspective of freely choosing individuals and the

[6] In my earlier exposition I rolled these two into one, though they are in principle distinct.

optimum positions they will voluntarily adopt. Consider, first, technological relations – relations between outputs and inputs. These are not given but constitute the solution to an economic problem. Even if factor supplies are fixed, the resulting relationship between goods produced (called by Samuelson the substitution or production indifference curve, perhaps following Lerner) is not given but must be derived. The derivation was kept in the background in Samuelson's (1938b), though in a 1941 article with Wolfgang Stolper, "Protection and Real Wages," a full visual derivation was given using a modified Edgeworth-Bowley box diagram, with the amounts of labor and capital used to produce each of two goods measured along the sides of the box and a locus of efficient production possibilities generated from the points of tangency of isoquants for each good (1941, 67-8). This derivation made explicit the important notion that the production possibilities curve is properly a frontier of optimum outcomes, respecting "certain equalities on the marginal physical productivities of non-specific factors" (Samuelson 1938b, 262).

With this issue settled and the appropriateness of optimizing terminology made plain, Samuelson moved quickly to complete his description of an optimum for an isolated individual. Such an individual will choose a preferred point on the production indifference (or possibilities) curve (frontier), a condition of equilibrium at this point being that the slope of the production curve will be the same as the rate of consumer substitution (ibid.).[7]

This was an early statement of the now-familiar first-order conditions for an optimum – two of them at any rate: (1) the marginal ratios of factor substitution must be equal, with factor rewards just equal to marginal physical product and covering the marginal

[7] Samuelson still eschewed indifferences curves, but he was willing to use small slope segments indicating the terms at which a consumption decision was made.

disutility of work (ibid., 262-63); and (2) the marginal rate of substitution in consumption (for the single individual involved) must be equal to the marginal rate of transformation between goods at the point of demand chosen (262).

Samuelson clearly considered the way now open for him to state his intended conclusion. Just as an isolated individual will choose to be at an optimum, so two individuals engaged in barter and each behaving as a price taker will end up somewhere on an Edgeworthian contract curve, the locus of points where the ratios of marginal utilities for all available goods are the same for both individuals (ibid., 265). This fulfills condition (3) in the usual list of first-order conditions.

The result, however, comes as something of a surprise. Two trading individuals will reach an optimum, but why is that joint optimum superior to the one reachable by each individual in isolation? Samuelson's answer is simply that each individual has chosen to trade, hence – an implicit appeal to the outcome of the argument in "A Note on the Pure Theory of Consumer's Behaviour" – the with-trade optimum must be considered by each to be a movement up their respective preference scales (ibid.). There is a telling image introduced that almost clinches the argument that an individual acting in isolation will achieve a "maximum position…such that a movement in any direction le[ads] to a less preferred situation, just as any movement from the top of a hill must be in a downward direction" (ibid., 264). But for this image to help here it must be the case that the joint optimum involves a jump to a higher peak, which however is just what Samuelson had not demonstrated, though it was implied. Leaving this aside for the moment, we must note an important clarification he introduced: the with-trade optimum is not a single point

but a locus (the contract curve), and the meaning of free trade equilibrium needs to be stretched to accommodate this fact (265).

At this stage in Samuelson's personal progress we have his argument from revealed preference, the implied visualization of the derivation of the production indifference curve (made explicit in Stolper and Samuelson (1941)), and the notion of equilibrium as optimum, with its requirements in terms of marginal equalities. We also have Samuelson's image of an optimum as a peak from which any movement must be down (and also down the preference scale). To this point, however, the superiority of free trade over autarky remained encrypted in what he would call in his next article in the sequence "a true maximum position." To be sure that he was sharing his insights effectively with those less familiar than he with Pareto optimum,[8] or just those of merely middle brow, this notion needed expressing in a number of alternative ways, each perhaps compelling to a different sort of reader.

The need for clarification was taken up in "The Gains from International Trade" (1939), in the *Canadian Journal of Economics and Political Science*. Here Samuelson started with a re-statement: for some prevailing configuration of prices, optimal quantities of goods and productive services are those which maximize for the economy as a whole the algebraic difference between total value of output and total factor cost. Trade is introduced by the simple device of supposing a new set of prices. As before, it is assumed to begin with that all individuals are alike. Samuelson then sets out to prove rigorously that every individual will be better off with some trade than at the prices prevailing in isolation. This amounts to showing that, if there is profit maximizing at the

[8] Samuelson was introduced to Pareto at Harvard as a member of the "Pareto Circle;" but the familiarity thus acquired was unusual among native English-speaking economists in the late 1930s.

new prices, then any situation involving those prices but the old quantities of goods and factor services must be less desirable.

In part Samuelson's argument here is a reprise of his 1938a ("A Note on the Pure Theory…"). If at the new prices (i.e., with trade) an individual is in a "true maximum position," then every prior situation must have been one in which the original quantities of goods could have been bought at the new prices and each productive service supplied in the original amounts, but for a lower algebraic sum. In other words, somewhat more of each good could have been bought and somewhat less of each service sold. But since no such alternative was chosen this proves that none of those possibilities was judged as desirable as the one chosen, the with-trade quantities at with-trade prices. The argument applies to each individual separately and thus to all, though if individuals are not all alike this gives rise to possibilities in which some gain and some lose with trade. Admitting this alters the formulation of what can be shown; what remains true is that winners could in principle bribe losers to accept the introduction of trade – Viner's novel statement concerning gains of 1937.

Samuelson complemented this still-cryptic attempt at a proof with a verbal argument exploiting the idea that the opening of trade brings with it a new profit-maximizing incentive to producers. At the new prices (with trade) producers can be assumed to strive to maximize the total value of output at those prices, given pre-assigned amounts of each productive factor. Samuelson takes it as too obvious to require spelling out that if trade is to represent an opportunity for betterment the with-trade terms of exchange must be better than they were before. Under that basic assumption, maximizing behavior by producers "will yield a larger money sum than any other production policy, and with a

larger sum of money more can be bought of every commodity than with a smaller one." This translates into the possibilities that more goods can be had with the same factor inputs or, that less productive services need be offered to obtain the same quantities of goods or, (a combination) somewhat more of every good can be had with a little less of each productive service (1939, 204-5).

Both forms of the argument here – that drawing essentially on revealed preference, and that resting on trade at better terms – were made without visual help. Cumulatively, to this point, visualization remained limited to the derivation of the production indifference curve in Stolper and Samuelson (1941), plus the image of optima as peaks in Samuelson's (1938b) article on welfare and trade. It is intriguing, then, that in a final paper on the gains issue, an article in 1962 in the *Economic Journal*, called "The Gains from International Trade Once Again," Samuelson's presentation was almost wholly visual.

In 1962 Samuelson formulated the proposition about gains that he thought it possible to prove, as: "free trade and ideal transfers could…give maximal world production" (1962, 821). The key element in establishing this, he emphasized, was that compensation or bribing did not have to be made out of specific fixed totals of goods, while with-trade goods totals – a set of possibilities – will be larger than in the absence of trade (ibid.). The 1938b assertion that both parties *are* better off under trade has become the more circumspect *could be made* better off, but the essential element, that more goods are available with trade, has not changed. What has altered is the manner of making the argument. Instead of index number formulations of choice possibilities and revealed preference, and in place of betterment possibilities implicit in trade's being a full

maximum position, we are now told that trade maximizes world production "in the sense of a *farthest out* world production possibility frontier" (1962, 829, emphasis added); alternatively, that "the trade frontier lies everywhere north-east of the autarky frontier" (821). North-east has replaced better; a simple, unambiguous visual directional indicator has been substituted for a complex and fraught term from welfare economics.

Samuelson's (1962) began with a diagram that could have been extracted from Viner's 1931 (and 1937) figure (fig. 3): a convex (from above) production locus and a with-trade price line, touching the production curve at one point. This figure illustrated consumption possibilities under autarky and free trade, where the terms of trade available are given and unalterable by the country in question. Such a country's consumption possibilities frontier with trade is defined by the terms of trade line, which lies everywhere north-east of the autarky frontier, except for the point of tangency, which Samuelson interprets as a limiting case in which trade neither hurts nor helps. With redistribution, moreover, this limiting case is avoidable. Notice that although positions north-east traditionally had been taken as unambiguously better within utility or indifference-curve analysis (as by Viner, though only for heuristic purposes), Samuelson makes no reference to utility or to indifference curves in his initial figure. For him the with-trade frontier simply shows that "society can have more of all goods (and less of all irksome inputs) with some trade" (1962, 821).

Samuelson next introduced a version of the same diagram but for the case where the country concerned can alter its terms of trade. He showed, borrowing an envelope curve construction due to Robert E. Baldwin, that here too the with-trade consumption possibilities frontier lies everywhere to the north-east of the autarky possibilities (except

for a point of tangency). The construction itself involves moving an offer curve, here used to show net imports at every point of equilibrium, around the production indifference curve.

The rest of Samuelson's 1962 article is taken up with refinements, the main one of which for our purposes has to do with the (ordinal) utility effects of re-distribution possibilities, which he had not previously treated, though he had been made aware of them as an undergraduate by Viner. At issue is whether zero deadweight redistributions will upset the initial conclusion, as would happen if the utility frontier for all redistributions possible of the with-trade consumption bundle (v, in fig. 4) curved inside (moved to the south-west of) those of the autarky consumption point (d, in fig. 4). Samuelson shows that while this might happen for these two (or any other) points taken pairwise, the envelopes of all such points must be such that the autarky utility envelope frontier, pq, lies uniformly (except for one touching possibility) inside that of the with-trade points, ef. With trade the pattern of imports will change, automatically moving v north-westward, so that it will come to lie north-east of point d and its redistribution curve d′dd″.

Figure 4, Samuelson's utility possibilities frontiers, with and without trade

[figure here]

Source: P.A. Samuelson, "The Gains from International Trade Once Again," *Economic Journal*, Dec. 1962, 824.

Some observations

I conclude with a number of observations that arise quite directly out of the two histories traced here – three if Viner is included. It will be clear in the way my comments are formulated that I am inferring and interpreting at the same time; they are expressed therefore in ways that leave them open to challenge.

1. First, visualization can come quite early or late in a process of analysis. Marshall's visual idea that the net benefit of trade is something akin to consumer's surplus, was already present when in the mid-1870s he set about writing his draft text for students on trade, subsequently abandoned, and decided that he wanted to keep his curves relating to domestic value because using offer curves to represent the "total benefit" from trade was not straightforward.[9] The mature benefit diagram that we find in *Money, Credit and Commerce* differs from a variant that survives from that early period only in its application of rectangulation to provide a composite linear measure of benefit.

Samuelson too began with and retained the idea that it could be shown that with trade all parties gain. This went through some refinements along the way, but in a sequence of attempts to clarify what was involved stretching over a quarter century visualization, as we have just seen, played a subsidiary role until the very end.

2. At the point where Samuelson opted for visualization seeing and understanding very obviously had become one: everything important could be "specified at a glance." In Marshall's case it was not clear – even to him – that he would ever achieve such a natural union with his chosen tools.

[9] Hence the otherwise slightly odd joint packaging of *The Pure Theory of Foreign Trade* with *The Pure Theory of Domestic Values* in Marshall's privately circulated (1879): see Whitaker (1975) I, 280, quoting Marshall.

3. The path to this natural union in Samuelson's case was not predictable, though it became clear to him how to effect it when he "saw" that a with-trade consumption possibilities frontier captures perfectly the idea of an optimum that he had been expressing algebraically and using the logic of revealed preference, in both cases probably too cryptically for many readers. But if it is not always possible to predict when visualization will come into its own, we can at least say in some cases that a path chosen will appear blocked. Marshall was aware that offer curves were not ideal for conveying the measure of net benefit from trade, but he found no alternative. Viewers too can sense that his way was blocked even without his confession to that effect; for there is just too much implicit in his net benefit diagram to allow it to live up to his own notion of what it is that makes a diagram superior to mathematics, that it can "present simultaneously *to the eye* the chief forces which are at work" (emphasis added).

4. Despite a lack of predictability in the Samuelson history, the path itself may be important to some readers. When I began teaching trade theory in the mid-1960s it was not at all obvious to me why Samuelson proceeded quite as he did in his 1962 article. Now, with a sense of the prior history (and an acquired knowledge of Viner) I understand better. At the time, however, I found it most helpful in fixing how gains arise simply to combine indifference curves with a production locus, at best a piece of sloppy shorthand that neither Viner nor Samuelson could have approved.

5. Samuelson is well known for his methodological pronouncement that mathematics, diagrams and words are alternative languages. However that may be, they are not equal on every occasion or to all people. He himself appears to have had relatively little need of visualizations; the optimum idea said most of what he needed for the task as he saw it,

and he created or picked up a diagrammatic derivation, or an image to illustrate a concept, only for particular subsidiary purposes along the way. But visualizations are of greater help to many people. Moreover, and in spite of what was said in the introduction about there being no core or universal quality in trade-related (or other) visualizations, it does seem to be the case that diagramming trade problems has a particular appeal. This is probably due to the low power of abstraction needed to shift from imagining two countries exchanging real goods to 2 x 2 x 2 models for demonstrating comparative advantage, the effect of trade on factor incomes, tariff problems and many others.

6. There is often some tension between such models and a desire to arrive at theoretically general results. Marshall understood this, and the tension is patent in the Samuelson story I have told here. But Samuelson also found a way to resolve it, which I presume is not a usual experience. When he began applying the Baldwin envelope and the utility possibility frontier to trade, precision was enhanced while exposition also became more straightforward. Samuelson found himself able to demonstrate mutual gain using two readily visualizable elements: (a) desired direction of change (north-easterly) and, (b) orientation (of the envelope/frontier curves): north-west to south-east, with no definite signs on the curvatures.

7. In this instance, where precision and simplification happily combined, the result was a more compelling statement. Marshall found himself on the opposite tack. In trying to make more precise his measure of the net benefit from trade he added layer upon layer to his basic visual device, piling the empirical triangle of surpluses onto the basic offer curve demonstration of how they might be thought of as arising, then adding rectangulation to the triangle to get a precise composite linear measure of these surpluses.

8. Visualization appears to be useful for various purposes. In our histories we have encountered it serving heuristic ends (Viner) as well as for demonstration (Samuelson's (1962) and Stolper/Samuelson (1941)). And of course it was used by Marshall to convey a measuring procedure. In this variety there may be a parallel to Mary Morgan's recently expressed idea that models in economics often serve purposes other than to demonstrate mechanisms, chief among them perhaps being that they serve as carriers of narratives (Morgan 2002).

9. I have drawn attention to the gap in Samuelson's attempts to articulate his notion of gains between 1939 and 1962. In a purely logical sense there was no gap; as Samuelson himself pointed out, "nothing in my 1939…required that the compensation or bribing be…out of fixed totals" (1962, 825). But this makes too light of the fact that great clarification was achieved in the interim, and of the fact that it was visual clarification. The crucial element in this phase of Samuelson's history was identifying the utility possibilities frontier as a device for showing outcomes without utility (measurement thereof) and for all redistributions. This frontier was developed by Samuelson, though not only by him, in the 1940s and 1950s (for a potted record see (1962), 823, note 4). Without the frontier device, Samuelson's 1939 article did what could be done, but it fell far short of that which could be visually comprehended in 1962. More revealing than arguing logical continuity would have been for Samuelson to acknowledge the visual transformation that had occurred in his own thinking about the problem. In fact he did just that, though in a curiously indirect manner which, however, also acknowledged where his own early efforts to explain the gains from trade came from: "Pareto's economics would have been better understood had he explicitly used the utility frontier

concept" (ibid .). Perhaps for a majority of those Samuelson was addressing in the years 1938-39, yes.

10. Audiences have been a controlling variable in my histories here, but not very visible; indeed, as just now, alluded to but scarcely examined. This is a weakness that only came to the fore when I began asking not only who was speaking but, to whom?

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