

**A Grin Without a Cat:
W. S. Jevons' Elusive Equilibrium**

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As in physical mechanics innumerable conditions may be comprehended under the principle of virtual velocity, so also there is a unifying principle in the mechanics of industry ... [which is] the law of supply and demand.

F. Y. Edgeworth

The notion of getting into equilibrium is a metaphor based on space to explain a process which takes place in time.

Joan Robinson

In the Preface to the first edition of his Theory of Political Economy (hereafter TPE), W. Stanley Jevons noted that, in chapter IV, he had "alluded" to the "cardinal difficulty with the whole theory" [Jevons 1871, p.x; 1970, p.45]. As Jevons did not make a habit of advertising the problems he recognised with his marginalist theory, the reference indicates that he considered the cardinal difficulty to be a matter of substantive concern. Because it was only alluded to, however, it was not clearly identified. The purpose of this paper is twofold. The first is to argue that the difficulty consisted in explaining how an equilibrium is attained in a market period and that Jevons was unable to do so. It has since been observed that the failure of post-classical theory¹ to provide "a rigorous basis for believing that equilibrium can be achieved or maintained if disturbed" is a "major lacuna in economic analysis" [Fisher 1987, p.26]. While that lacuna was evident to Jevons, the second purpose of the paper is to show how he obscured both the presence and the significance of the problem with a series of rhetorical devices in TPE.

The core of Jevons' difficulty turned on the relationship between the marginal utility theory and "the laws of supply and demand" in a market period. One of Jevons' principal arguments in TPE was that, with the utility theory, he had, for the first time, provided a rigorous theoretical explanation for the laws that, as "facts", recorded "actual" market phenomena. This meant, in turn, Jevons claimed, that the theory was "verified" in the functioning of actual markets [Jevons 1970, pp.43,148]. To explain the attainment of equilibrium with the theory, Jevons depicted a representative

¹ Often referred to as neoclassical theory. Partly for the reasons given by Tony Aspromourgos [Aspromourgos 1986], the term post-classical is more appropriate.

transactor adjusting purchases in a sequence of disequilibrium transactions with a given exchange ratio (or market price). The problem was that, for Jevons, the laws of supply and demand entailed that every transaction would change the price. The attainment of equilibrium could not, therefore, occur as any transaction would change one of the data of the equilibrium position. Unable to solve the problem, Jevons produced a confusing account of market-period trading in TPE.² On the one hand, he gave the impression at a number of places that he could explain how equilibrium was attained through adjustments with disequilibrium transactions. At the same time, he alluded to the problem, but failed to clearly identify it. He did not make clear, therefore, that his account was restricted to the depiction of equilibrium outcomes, the attainment of which he could not explain. He then resorted to the argument that all market transactions could be depicted as occurring at equilibrium and attempted to defend that result, principally by drawing on a number of arguments from mechanics and natural philosophy (physics). Jevons had initially developed his marginalist analysis using a series of mechanical metaphors. The basic exchange equilibrium condition, for example, was derived from that for a lever in static equilibrium. In explaining that derivation in TPE, Jevons defended his analysis by referring to the principle of virtual velocities from (variational) mechanics, which argued that all outcomes in a system could be explained in terms of equilibrium. It was consistent with his only alluding to the attainment problem, however, that the significance of the principle of virtual velocities was not clearly explained.

The analysis is presented in five sections. The first considers Jevons' discussion of market-period trading in Chapter IV of TPE, where he argued that the marginalist theory was practically verified in 'actual' markets. Section 2 then explains his difficulty with equilibrium. The following section discusses the passages where Jevons alluded to the problem, while section 4 considers the lever analogy and the reference to the principle of virtual velocities. It is concluded that Jevons' confusing discussion of his theory and its relevance for explaining events in actual markets was

² Chapter IV of TPE is concerned with a market-period analysis, where the stock of commodities to be traded is given at the start of each period. Chapter V set out a long-period analysis where, with changes in production levels, market-period prices fluctuate around or tend toward, a long-period position, characterised in terms of the cost of production. Jevons did not explain this periodisation (which entails his use of the classical method) in TPE and the analysis is also difficult to understand because of his peculiar definition of cost of production. For discussion, see White 2003a; 2003b.

a deliberate rhetorical strategy, designed to avoid a clear discussion of his equilibrium problem. Section 5 shows how the context in which TPE was written explains why Jevons adopted that strategy.

1. Market-Period Trading

Jevons' discussion of trading (or exchange) in a market period in Chapter IV of TPE assumed a "theoretically perfect" market which was characterised by six conditions:

- (i) the ratio of exchange "between any two persons should be known to all the others";
- (ii) transactions were motivated purely by self-interest and the maximisation of utility;
- (iii) "perfectly free competition" prevailed so that "anyone will exchange with anyone else";
- (iv) there were no conspiracies for withholding supplies to increase prices;
- (v) commodities were homogeneous;
- (vi) there was a single price (or ratio of exchange) "at any moment"

[Jevons 1970, 133-4,136,137].³

The uniform price was "a general law of the utmost importance in economics", which, in the second edition, was labelled "the law of indifference ... in the same open market, at any moment, there cannot be two prices for the same kind of article" [Jevons 1970, p.137].⁴ The archetypal market used to illustrate the argument was that for wholesale commodities, such as cotton and coal, or financial instruments, such as stocks [ibid. p.132]. In those markets, Jevons asserted, brokers established the prevailing price: "The theoretical conception of a perfect market is more or less completely carried out in practice. It is the work of brokers in any extensive market to organize exchange, so that every purchase shall be made with the most thorough acquaintance with the conditions of the trade ... It is only thus that a definite market price can be ascertained at every moment, and varied according to the frequent

³ Jevons' use of the calculus also required that commodities were infinitely divisible. The difficulties this created for his analysis, put to one side here, are considered in White 2001.

⁴ In the first edition, the law was referred to as the principle of uniformity [Jevons 1871a, p.99]. All emphases in material cited here appear in the original texts.

news capable of affecting buyers and sellers" [ibid. pp.133,134].⁵ This meant that constantly changing prices were consistent with the law of indifference: "Though the price of the same commodity must be uniform at any one moment, it may vary from moment to moment, and must be conceived as in a state of continual change" [ibid. p.137]. The key analytical role attributed to brokers was further illustrated when Jevons, in effect, defined a market in terms of the law:

By a market I shall mean two or more persons dealing in two or more commodities, whose stocks of those commodities and intentions of exchanging are known to all ... It is only so far as this community of knowledge extends that the market extends. Any persons who are not acquainted at the moment with the prevailing ratio of exchange, or whose stocks are not available for want of communication, must not be considered part of the market [ibid. p.133].

It was a peculiar argument. With no trading within a spread of buy and sell offers by transactors, the maximisation assumption entailed that all trades took place at equilibrium. Yet there was no explanation for how brokers would establish a uniform price and certainly not at every moment of trading [Fusfeld 1990].⁶ The figure of brokers thus acted as a deus ex machina, analogous to that of the auctioneer in later post-classical theory. It should be noted, however, that Jevons' notion of perfect competition was quite different from that of the later theory in two important respects. The first was that, contrary to some accounts,⁷ he did not depict all transactors as price-takers in TPE. Attributing a price-taking model to TPE ignores the role of brokers and Jevons' clear statement that, with continuous trading in a market period, any set of trades would change the price: "In theory, this effect of exchange upon the ratio of exchange must be conceived to exist in some degree, however small may be the purchases concerned" [Jevons 1970, p.138]. The second difference with the later

⁵ It has been argued that Jevons regarded the law of indifference as "an abstraction from reality. In reality, differences in observed prices arose all the time in practice..." [Peart 1998, p.310]. This depends, in part, on reading Jevons' statement that theoretically perfect markets existed "more or less completely in practice" to mean that perfect markets rarely occurred in reality [ibid. p.316].

⁶ The analysis also erased the important role in British stock exchanges of jobbers (or dealers) who carried out brokers' orders for buying and selling [Attard 2000].

⁷ Creedy 1986, pp.45,46; Schabas 1990, p.33; Peart 1998, pp.308, 309.

theory concerned the knowledge of transactors. Jevons defined a market as "theoretically perfect only when all traders have perfect knowledge of the conditions of supply and demand and the consequent ratio of exchange" [ibid. p.134]. This perfect knowledge, however, entailed that participants had that information at any moment and not that they had complete knowledge of future events, even on the same day.⁸ Jevons made this clear in a note added to the second edition when, discussing the law of indifference and how the theory was "verified in practice", he referred to

New York stock markets, where it is the practice to sell stocks by auction in successive lots, without disclosing the total amount to be put up. When the amount offered begins to exceed what was expected, then each successive lot brings a less price, and those who bought the earlier lots suffer. But if the amount offered is small, the early buyers have the advantage. Such an auction sale only exhibits in miniature what is constantly going on in the markets generally on a large scale [ibid. p.137n].

Jevons' statement that perfect markets were to be found more or less completely carried out in practice, indicates that he acknowledged there were some exceptions. Four can be identified in TPE, two of which concerned traders in specialised commodity or financial markets. The first concerned indivisible and heterogeneous commodities for which there were a small number of transactors [White 2001]. The second was that speculators could manipulate market outcomes through, for example, the use of corners. Changes in prices would then bear "no proper relation to the existing supplies", producing "unnatural ratios of exchange". The solution was state intervention in the form, for example, of regulations requiring information to be posted regarding the amount of stocks to be traded on a particular day [Jevons 1970, pp.133,134].⁹ The third exception concerned the law of indifference when Jevons

⁸ Cf. the unqualified assertion that Jevons' theory assumed "perfect competition and perfect information" [Schabas 1990, p. 40].

⁹ It has been suggested that Jevons' reference to speculators "meant that trades would occur that violate his equilibrium conditions for exchanges" [Peart 1998, p.316]. The violation, however, concerned one of the conditions for a perfect market. The equilibrium conditions for exchange would still apply, as they simply entailed that purchases were made at the price at any moment. Such prices would be "unnatural" because they would be higher than those in a perfect market.

referred to “Such differences as may practically occur arise from extraneous circumstances such as the defective credit of purchasers, their imperfect knowledge of the markets, and so on” [ibid. p.137]. This was, however, the only acknowledgement Jevons made in TPE regarding such divergences. While John Stuart Mill and J.E. Cairnes regarded price dispersion as the rule in retail (but not wholesale) markets,¹⁰ Jevons treated dispersion as an exception which required little attention.

The fourth exception to the conditions required by the theory concerned the behaviour of final consumers. At a number of points in TPE, Jevons discussed whether all "individuals" would behave as required by the theory in adjusting their purchases in a continuous (marginalist) maximising manner as prices changed [Jevons 1970, pp.86,108-9,119,135-6]. He acknowledged that some ("many" at one point [ibid. p.86], although he usually referred to one or a few individuals) would not do so: “We cannot usually observe any precise and continuous variation in the wants and deeds of an individual, because the actions of extraneous motives, or what would seem to be caprice, overwhelms minute tendencies”. However, because he assumed that behaviour was normally distributed (according to the law of error) purchasers would, on average, behave in the requisite manner. So, for example, if the price of a commodity increased,

a single individual ... probably continues his ordinary consumption until accident directs his attention to a rise in price, and he then, perhaps, discontinues the use of the article altogether for a time. But the aggregate or what is the same, the average consumption of a large community will be found to vary continuously or nearly so. The most minute tendencies make themselves apparent in a wide average. Thus, our laws of economics will be theoretically true in the case of individuals, and practically true in the case of large aggregates; but the general principles will be the same, whatever the extent of the trading body considered [ibid. p.135].

Indeed, the way in which particular individuals behaved was virtually irrelevant: "the movement of trade and industry depends on averages and aggregates, not on the whims of individuals" [ibid. p.136]. Due to the law of error, “accidental and disturbing

¹⁰ See the discussion and references in White 1994b, p.152.

causes will operate, in the long run, as often in one direction as the other, so as to neutralise each other" [ibid. p.86]. The use of that assumption entailed that consumers' behaviour was independent, which marked a significant break with the interdependent behaviour assumed by earlier economists such as Adam Smith and Nassau Senior.¹¹

The effect of Jevons' discussion of possible exceptions to perfect markets was to reinforce his claim that such markets could be taken as the rule. The law of indifference was crucial because it governed the possibility of equilibrium trades and hence the material welfare maximisation results that, Jevons claimed, followed from "perfect freedom of exchange":

so far as is consistent with the inequality of wealth in every community, all commodities are distributed by exchange so as to produce the maximum of benefit ... No one is ever required to give what he more desires for what he less desires, so that perfect freedom of exchange must be to the advantage of all [Jevons 1970, p.171; see also p.173].

It was, however, in explaining the attainment of an equilibrium position that Jevons realised he faced the cardinal difficulty with the whole theory.

2. The Equilibrium Problem

The analysis of market-period trading in chapter IV of TPE was designed to show that the facts of the laws of supply and demand could be explained by the marginal utility theory. For two representative transactors trading commodities x and y , the basic equilibrium exchange condition was $\phi_x/\psi_y = dy/dx = y/x$, where: ϕ_x/ψ_y is the ratio of the final degree of utility for the two commodities; dy/dx is the marginal exchange ratio; y/x is the market exchange ratio. In Chapter V of TPE, Jevons added a

¹¹ It has been suggested that "Jevons sometimes presumed that the effects of 'caprice' would balance across individuals, so that on average, an aggregate specification will be correct" [Peart 1998, p.321, n.10; see also p.319]. The point, however, is not that Jevons "sometimes presumed" this, but rather that it was at the core of his argument for the validity of the theory. Milgate [1987, p.180] confuses Jevons' reference to "the long run" in that context [Jevons 1970, p.86] with a discussion of long-period equilibrium, involving production.

price ratio for the two commodities to the equilibrium condition, so that $y/x = p_x/p_y$ [Jevons 1970, pp.141-4, 204]. As has often been noted, this is basically the same condition for consumer equilibrium that can be found in today's microeconomics textbooks.¹² In Chapter IV, Jevons argued that, with the market exchange ratio given at any moment by the law of indifference, the theory depicted an equilibrium between supply and demand:

We may regard x as the quantity demanded on the one side and supplied on the other; similarly, y is the quantity supplied on the one side and demanded on the other. Now, when we hold the two equations to be simultaneously true, we assume that the x and y of one equation equal those of the other. The laws of supply and demand are thus a result of what seems to me the true theory of value or exchange [Jevons 1970, p.144].

In the section of the chapter immediately preceding that analysis, Jevons had discussed how the equilibrium maximisation point could be reached, using a utility diagram for a representative transactor [Jevons 1970, p.140].

Figure 1

The transactor held a stock of commodity b (the supply), exchanging units of it for those of commodity a (the demand). The final degree of utility function for a (pr), was "measured" from left to right. The corresponding function for b ($p'r$), reversed and superimposed on that for a , was measured from right to left. Acquisition of the unit aa' would produce a net utility gain ($a'd > ac$) and exchange would continue until the equilibrium point q . Transactions would not persistently take place to the right of q as there would be a net utility loss ($eb' < fb$). In representing disequilibrium transactions, the diagram showed how utility was maximised at equilibrium: "Exchange will thus go on until each party has obtained all the benefit that is possible, and loss of utility would result if more were exchanged. Both parties, then, rest in satisfaction and equilibrium, and the degrees of utility have come to their level, as it were." It is important for the

¹² The textbook version of the equilibrium condition with the (Hicks-Allen) geometry of indifference curves and budget lines is used to construct a theory of market demand. This may have encouraged the myth, begun by Alfred Marshall, that Jevons emphasised demand rather than supply in TPE.

analysis below to note that, although Jevons acknowledged that the theory dealt with "infinitely small quantities" of commodities, his discussion also referred to finite quantities, such as ten pounds of corn [Jevons 1970, p.140].¹³

The depiction of adjustment to equilibrium with a given exchange ratio appears, at first sight, to be consistent with references, elsewhere in the same chapter, to "the conditions of a great market where vast quantities of some stock are available, so that any one small trader will not appreciably affect the ratio of exchange". In such cases, the equilibrium condition "accurately represents the position of an individual consumer with regard to the aggregate trade of a large community, since he must buy at the current prices, which he cannot in any appreciable degree affect" [ibid. pp.150,151]. The argument was, however, misleading. As was noted above, Jevons made clear that any set of trades, represented by the behaviour of a representative transactor, would change the ratio and, therefore, "the conditions of utility" [ibid. p.137]. This resulted in Jevons' difficulty: adjustment towards an equilibrium in a series of disequilibrium transactions would induce a parameter shift (the price) and hence a new equilibrium position. Any convergence to an equilibrium through disequilibrium transactions would be akin to chasing a will o' the wisp. Nor was the problem restricted to transactions by a group of traders. As was noted above, Jevons argued that, in theory, any trade would change the ratio. Even when he referred to an individual consumer whose transactions could not appreciably affect the price (see above), he acknowledged in the same

¹³ The single diagram in TPE can be compared with the more complicated series used in 1868 when Jevons discussed the exchange theory with H.C. Fleeming Jenkin [Black 1973-81, iii, pp.166-78]. If the diagrams in Jenkin's letters are a reliable guide, Jevons drew intersecting utility curves for different transactors. This would help explain his subsequent comments about priority over Jenkin in using intersecting curves to illustrate the determination of market price (see the discussion and references in White 1989, pp.443-8). The single-transactor diagram in TPE appears to have been used for simplicity of exposition. In Jenkin's letters, a single diagram for two transactors could only represent trade in one commodity. Trade with two commodities, as required by the exchange theory, would necessitate at least one other diagram or a series of diagrams. The single-transactor diagram in TPE could show two commodities and hence illustrate the basic equilibrium exchange condition. It was, however, limited, which would explain Jevons' comment that the theory could not be "completely" represented by the diagram [Jevons 1970, p.141]. Jevons' subsequent claim that he had drawn diagrams to illustrate the determination of market price was misleading, as he assumed a given price for each set of transactions.

paragraph that "the smallest purchasers do affect the market in some degree" [ibid. p.151].

One possible solution to the problem can be found in Jevons' discussion, elsewhere, of the properties of the laws of supply and demand. Although he insisted that the marginalist theory provided the analytical basis for the laws, he did not explain their meaning in TPE. An account can be found, however, in the record of his mid-1870s lectures at Owens College [Black 1973-81, vi, pp.81-2], the Political Economy primer [Jevons 1878, pp.99-100] and the unfinished Principles of Economics [Jevons 1905, ch.9]. The laws were summarised in the primer as follows: "a rise of price tends to produce a greater supply and a less demand; a fall of price tends to produce a less supply and a greater demand. Conversely, an increase of supply or a decrease of demand tends to lower price, and a decrease of supply or an increase of demand to raise price" [Jevons 1878, p.99]. For Jevons, the laws were empirical phenomena or facts that were the result of a complex of causal factors in market trading. Because of that complexity, there was little to be gained analytically in drawing supply and demand functions as the facts were known [White 1989; 2003b]. Jevons argued, however, that it was possible, in principle, to explain the primary factors driving transactors' behaviour by the marginal utility theory, assuming that other factors could be put to one side.¹⁴ In terms of Figure 1, changes in demand and supply with the respective price and quantity outcomes could be represented and explained by shifts in the utility functions. The difficulty was, however, to explain how an equilibrium could be attained.

In the Principles, Jevons argued that the laws "are in no sense ultimate, natural, or invariable laws, but only expressions of the general course of phenomena exhibited in commerce when there are many buyers and sellers". The market characteristics necessary for the laws were the same as those for perfect competition in TPE [Jevons 1905, pp.56,57]. Consistent with TPE, the summary of the laws indicated that any change in demand or supply would change the price, producing an equilibrium: "The whole problem of value is supposed to be summed up in this equation, that the value will be adjusted to the point at which the quantity demanded is equal to the quantity offered" [Black 1973-81, vi, pp.81-2; see also Jevons 1878, pp.99-100]. The record of

¹⁴ The other factors included speculation, the distribution of income and wealth (demand) and transaction costs (supply).

the Owens College lectures acknowledged that it was difficult to explain how the equation of demand and supply occurred, "except the higgling of the market achieves it" [Black 1973-81, vi, p.82]. While the primer also referred to higgling as settling the price at a "Butter Cross" in a "market town", Jevons claimed there that all transactions took place at equilibrium:

The [market] price must be such that the quantity demanded at any time is equal to the quantity supplied.

The market price will be such that the demand at that price will equal the supply at that price. The quantity of butter or any other commodity that is sold must equal what is bought, because it is not sold until it is bought; but the price will settle itself accordingly [Jevons 1878, p.100].

This was a peculiar claim - all transactions took place at equilibrium because, by definition, what was bought must equal what was sold. Here, Jevons was repeating an argument made by John Stuart Mill in 1869 when he rejected part of William Thornton's critique of the role of the laws of supply and demand in explaining price formation. In his On Labour (1869), Thornton had argued that, on a market day, most trading took place at disequilibrium positions. Disequilibrium was indicated by unsold inventories that provided a signal for sellers to adjust prices in a series of steps. Reviewing Thornton's text, Mill claimed that inventories should not be regarded as part of the market supply, an argument which, in effect, made demand and supply equal for all trades. As J.E. Cairnes explained to Mill, that explanation for equilibrium was a tautology (a "truism"), a point Mill conceded.¹⁵

Jevons also appeared to refer to Mill's tautology in TPE when claiming that "it is not difficult to find practical illustrations which will show how [the marginalist theory of exchange] is verified in the actual working of a great market. The ordinary laws of supply and demand, when properly stated, are the practical manifestation of the theory" [Jevons 1970, p.148]. Referring to Thornton's critique and to Mill's review, Jevons claimed (quite misleadingly) that he not only accepted Thornton's "view" but would "carry it further":

¹⁵ See the discussion and references in White 1994b, pp.161-2 and section 5 below. Mill's attraction to a tautological treatment of equilibrium was evident from the 1830s in some notes he made on Nassau Senior's Outline [Mill 1945, p.134].

The [marginalist] theory consists in carrying out this view to the point of asserting that it is only comparatively insignificant quantities of supply and demand which are at any moment operative on the ratio of exchange. This is practically verified by what takes place in any very large market – say that of the Consolidated Three Per Cent Annuities ... Now the theory [asserts] ... that the market price of the funds is affected from hour to hour not by the enormous amounts which might be bought or sold at extreme prices, but by the comparatively insignificant amounts which are being bought or sold at the existing prices. A change of price is always occasioned by the overbalancing of the inclinations of those who will or will not sell just about the point at which prices stand. When Consols are at 93^{1/2}, and business is in a tranquil state, it matters not how many buyers there are at 93, or sellers at 94. They are really off the market [Jevons 1970, pp.149-50].

In the context of the references to Thornton and Mill, this statement had the effect of claiming that stocks could be ignored when explaining an equilibrium between demand and supply. If that was consistent with the definition of equilibrium in his primer,¹⁶ Jevons had also confirmed that comparatively insignificant transactions would change the price. However, he then gave the impression that the theory could deal with disequilibrium transactions:

When the price of funds is very steady and the market quiescent, it means that the stocks are distributed among holders in such a way that the exchange of more or less at the prevailing price is a matter of indifference.

In practice, no market ever long fulfils the theoretical conditions of equilibrium, because, from the various accidents of life and business, there are sure to be people every day compelled to sell, or having sudden inducements to buy. There is nearly always, again, the influence of prospective supply or demand, depending on the political intelligence of the moment. Speculation complicates the action of the laws of supply and demand in a high degree, but does not in the least degree arrest their action or alter their nature. We shall never have a science of economics unless we learn to discern the operation

¹⁶ See also Jevons' reference in TPE to Mill's discussion of equilibrium having the appearance of a 'truism' [Jevons 1970, p.143].

of law even among the most perplexing complications and apparent interruptions [Jevons 1970, p.150].

It might be suggested that the ambiguous statements regarding price-taking and the possibility of accounting for disequilibrium transactions simply reflected confusion on Jevons' part. There are, however, two reasons for concluding that he was fully aware of the equilibrium attainment problem. The first is that, when he referred to the cardinal difficulty with the whole theory in the Preface to the first edition, Jevons wrote that he had alluded to the problem in the section of Chapter IV on the "Ratio of Exchange" [Jevons 1871a, pp.91-5], which was retitled "The Law of Indifference" in the second edition [Jevons 1970, pp.136-9]. In that section, having discussed the law of indifference and how prices "must be conceived in a state of continual change", Jevons added: "Theoretically speaking, it would not usually be possible to buy two portions of the same commodity successively at the same ratio of exchange, because no sooner would the first portion have been bought than the conditions of utility would have been altered". He reiterated the point in the same paragraph: "In theory this effect of exchange upon the ratio of exchange must be conceived to exist in some degree, however small may be the purchases made" [ibid. pp.137,138]. This seems to be a reference to the attainment problem. It remained an allusion, however, because Jevons did not make clear to the reader that the point entailed that he could not explain how an equilibrium was attained.

The second reason for concluding that Jevons was aware of the attainment problem was the way he proceeded, in the same section, to defend his basic equilibrium exchange condition against the criticism that it was unsatisfactory because it was not possible to derive a total integral of utility from it [Jevons 1871a, pp.93-5; 1970, pp.138-9]. Following a restatement of that critique by a reviewer of the first edition, Jevons added a further defence in a new section in the second edition [Jevons 1970, pp.144-7]. That discussion was clear in that the problem (the absence of integration) was identified and a defence provided. It does not appear, therefore, that Jevons was referring to the integration criticism when he wrote of alluding to the central difficulty in the 1871 Preface.¹⁷

¹⁷ One commentary, which assumes that the allusion did refer to integration [Schabas 1990, pp.34,40,42,52], suggests that Alfred Marshall agreed with the integration criticism [ibid. p.151, n.31]. That was not the case [White 2001].

The integration criticism was quite distinct from the attainment problem in that the latter was pertinent with or without integration. Jevons' defence of the integration criticism is important to consider here, however, because of its implications for his difficulty. His defence turned on using metaphors from mechanics and natural philosophy. That is not surprising, as the discussion in the second edition made clear how the basic equilibrium condition was derived from that for a lever in static equilibrium. The significance of the metaphors was not simply that they provided a defence against the integration criticism but, more importantly, that they also provided a rationale for treating all trades as equilibrium transactions in a market period. It will be convenient to discuss Jevons' comments in the first edition of TPE in section 3. The following section will consider the further defence added to the second edition.

3. Statics and Dynamics

When Jevons referred to "the cardinal difficulty with the whole theory" in the Preface to the first edition of TPE, he added that, "So able a mathematician as my friend Professor Barker, of Owens College, has had the kindness to examine some of the proof-sheets carefully; but he is not, therefore, to be held responsible for the correctness of any part of the work" [Jevons 1970, p.45]. The reason for the qualification was that Barker, professor of mathematics at the College, objected to Jevons' basic conditions for equilibrium in exchange. As Jevons subsequently noted in a letter to George Darwin, Barker insisted that the conditions "are (or at any rate ought to be) differential equations demanding integration, whereas I hold that, though deduced by the use of differentials, they are simply algebraic equations. The problem, as I regard it, is a [statical] one" [Black 1973-81, iv, p.87].¹⁸ Harriet, Jevons' wife, later recalled a sharp disagreement on the matter: "Mr. Jevons showed him several of the proofs and I remember quite well the argument they had over one page, but Mr.

¹⁸ In the version of this letter published by Harriet Jevons in the Letters and Journal [Jevons 1886, pp.327-8], the word "statistical" appears, rather than "statical". In reprinting the letter in the Papers and Correspondence, Professor Black [1973-81, iv, pp.87-8] suggests that was a misprint, given Jevons' statements in TPE. The problem may have been the transcription of the letter. The same point could also be made about the word "differential" after the close of the bracket in the quotation above. This appeared as "different" in the Letters and Journal and was silently corrected in the Papers and Correspondence.

Jevons said he could not put it otherwise than he had, nor could he see that it was wrong" [Konekamp 1982, p.398].¹⁹

In the first edition, Jevons' discussion of his "ratio of exchange" as a "differential coefficient" turned on the difference between statics and dynamics used in mechanics as he outlined his defence in three steps. The first was to acknowledge that his approach was limited in that the mathematics could not depict the dynamics or motion of the system:

The real condition of industry is one of perpetual motion and change. Commodities are being continually manufactured and exchanged and consumed. If we wished to have a complete solution of the problem in all its natural complexity, we should have to treat it as a problem of motion – a problem of dynamics [Jevons 1970, p.138].²⁰

This was confusing. Chapter IV examined a market period with a parametric commodity stock at the beginning of the period. The reference here to production, exchange and consumption referred to an analysis of a series of market-periods tending toward, or oscillating around, a long-period equilibrium position as in Chapter V [White 2003b]. The "complete solution" would presumably entail a mathematical analysis of the adjustment toward the long-period equilibrium. Jevons noted, however, that such an analysis was not "within our power" [Jevons 1970, p.138].

The second step was to acknowledge that the analysis could, therefore, deal only with statics:

It is only as a purely statical problem that I can venture to treat the action of exchange. Holders of commodities will be regarded not as continuously

¹⁹ Barker's particular objection [Konekamp 1982, p.398] was to the statement of the equilibrium condition as $\Delta y/\Delta x = y/x$ or $\Delta y = y/x \cdot \Delta x$ [Jevons 1871a, p.99; 1970, p.141]. The same criticism was made in an unsigned review of the first edition published in the Saturday Review [Black 1973-81, vii, pp.152-7].

²⁰ Was the reference to "perpetual motion", in the first sentence of this quotation, a joke? It was a standard argument at the time that machines were not, in themselves, a source of power and could only transmit force, so that the notion of a perpetual motion machine was a delusion.

passing on these commodities in streams of trade, but as possessing certain fixed amounts which they exchange until they come to equilibrium [ibid. p.138].

With the reference to transactors possessing certain fixed amounts, this step switched the argument to that for a market period. The static analysis of exchange included the process of adjustment to equilibrium. It appeared, therefore, to allow for the possibility of disequilibrium transactions. The final step, however, indicated that an adjustment process was precluded:

It is much more easy to determine the point at which a pendulum will come to rest than to calculate the velocity at which it will move when displaced from that point of rest. Just so, it is a far more easy task to lay down the conditions under which trade is completed and interchange ceases, than to attempt to ascertain at what rate trade will go on when equilibrium is not attained [ibid].²¹

It was on this basis that Jevons justified his use of a differential coefficient equilibrium condition. If the analysis depicted dynamics, he would require "differential equations, which would have to be integrated". That was not necessary, however, "in the statical view of the question" [ibid. p.138]. Given the law of indifference, "the last increments in an act of exchange must be exchanged in the same ratio as the whole quantities exchanged". With infinitely small units of a commodity, all transactions would take place at a (momentary) exchange ratio [ibid. p.139].

It was a confusing discussion. Shifting, without clarification, from a long period to a market period, the upshot of the discussion was that the mathematical analysis could not depict transactors "possessing certain fixed amounts of commodities which they exchange until they come to equilibrium", as Jevons had claimed in the second step of his argument. Instead, the analysis was consistent only with the equilibrium condition where "trade is completed and interchange ceases". The reader, however, was given the impression that the analysis could deal with disequilibrium trades, because the next section of the chapter contained Jevons' diagram of crossed and superimposed utility functions for a single transactor with the discussion of how exchange would continue until equilibrium was attained (see above). That was followed with a section on "The Symbolic Statement of the Theory" [Jevons 1970, pp.141-4] where, criticising Mill's

²¹ See further discussion of the pendulum metaphor in section 4 below.

analysis of supply and demand, Jevons claimed that "our theory is perfectly consistent with the laws of supply and demand" [ibid. p.143]. Exchange was depicted in terms of an equilibrium condition for two transactors. At one point, however, Jevons acknowledged that each transactor "must ... derive equal utility from the final increments, otherwise it will be for his interest to exchange either more or less, and he will disturb the conditions of exchange" [ibid. p.142]. Once again, he had alluded to the attainment problem but failed to explain its significance for his claim about the analytical compatibility between the utility theory and the laws of supply and demand.

4. Virtual Realities

In the Preface to the first edition of TPE, Jevons noted that his

theory of economy ... presents a close analogy to the science of statical mechanics, and the laws of exchange are found to resemble the laws of equilibrium of a lever as determined by the principle of virtual velocities. The nature of wealth and value is explained by the consideration of indefinitely small amounts of pleasure and pain, just as the theory of statics is made to rest upon the equation of indefinitely small amounts of energy [Jevons 1970, p.44].

In the second edition, when defending the basic equilibrium exchange condition against the integration criticism, he explained the analogy in some detail [ibid. pp.144-7], referring to the following diagram from one of the "best modern elementary" treatments of mechanics, Philip Magnus' Lessons in Elementary Mechanics [ibid. p.144; Magnus 1875, p.128].

Figure 2

AB is a lever turning, without friction, about the fulcrum C. P is a force applied at A, while W is the force exerted (or resistance overcome) at B. If the lever turns through the arc ACA', the work done by P = P.AA' and the work done by W = W.BB'. For equilibrium, the work is equivalent, such that P.AC = W.BC, or $W/P = AC/BC$. Hence, $W/P = AA'/BB' = AC/BC$. The ratio of the point masses (W/P) is inversely proportional to both the ratio of the arcs of displacement (AA'/BB') and the ratio of point distances from the fulcrum (AC/BC). With an infinitesimally small displacement,

the equilibrium condition was "exactly similar in form to ... [that for] the theory of value" [Jevons 1970, p.145], so that

$$\begin{aligned} W/P &= AA'/BB' = AC/BC \\ \phi x/\psi y &= dy/dx = y/x \end{aligned}$$

The final degrees of utility corresponded to the forces, the marginal exchange to the displacement arcs and the exchange ratio to the distance of the point masses from the fulcrum (see Jevons' diagram, *ibid.* p.146).

Jevons' familiarity with the theory of the lever, used as the basic theoretical explanation for a balance, was the result of his training and work as a chemist and, especially, as a gold assayer at the Sydney Mint while in Australia (1854-59). Utilising that experience, at the beginning of 1861 he began work on a number of entries for a Dictionary of Chemistry, the first of which, "Balance", provided a detailed examination of the practice of using balances in chemistry, coupled with an explanation of the theory of the balance in terms of a lever [Jevons 1863].²² The entry opened as follows:

Chemistry being concerned with the relative masses or quantities of the elements which compose all known substances, and the weight or force of gravitation of a body being the only practicable measure of its mass or quantity of matter, the balance, which shows the equality of two weights, and may hence determine the ratio of all commensurable weights, is the chemist's most important instrument [Jevons 1863, p.481].

Jevons had derived his marginalist theory of behaviour in 1860 from the representation of behaviour as a mechanics of balancing gravitational forces in Richard Jennings' Natural Elements of Political Economy (1855) [White 1994a]. The quotation above suggests how he was then able to both adapt and extend Jennings' analysis in formulating the theory of exchange with the lever analogy. The discussion of the balance in TPE was, however, different from that in the chemistry Dictionary in two important respects. The first was Jevons' reference to "indefinitely small amounts

²² See also the discussion in Maas 2001, chapter 7, which considers Jevons' more general use of the balance metaphor in his political economy.

of pleasure and pain" as akin to "indefinitely small amounts of energy" [Jevons 1970, p.44]. This was in line with Magnus' discussion [Magnus 1875, p.125] but had only become part of Jevons' language from the mid-1860s [White 2003a]. The second and more important difference here was the use of the principle of virtual velocities. Jevons argued that, although it was "disguise[d]" in "elementary works", the principle was "the real foundation of the science" of mechanics. He summarised it as follows:

if any number of forces be in equilibrium at one or more points of a rigid body, and if this body receive an infinitely small displacement, the algebraic sum of the products of each force into its displacement is equal to zero. In the case of a lever of the first order, this amounts to saying that one force multiplied into its displacement will be neutralized by the other force multiplied into its negative displacement [Jevons 1970, p.145].

Without further explanation, it might seem that this was much the same argument as in the chemistry Dictionary, where the condition of static equilibrium for a balance was explained as "the sum of the moments of forces on one side [is] equal to that on the other" [Jevons 1863, p.487]. The only difference might appear to be that the argument in TPE depended on infinitesimally small changes, whereas the Dictionary entry (and, as Jevons noted in TPE, Magnus' account) dealt with finite changes. The principle of virtual velocities, however, was a general balancing principle that was supposed to apply to all mechanical systems.

Jevons' initial formulation of the balance metaphor drew on two principal theoretical sources. The first was the treatment of the calculus by Augustus De Morgan, whose classes Jevons had attended at University College, London, when he returned from Australia in 1859 [Black 1973-81, ii, p.403].²³ The second was S.D. Poisson's Treatise on Mechanics (1833), an influential exposition of the French tradition of variational mechanics which, in referring to the work of J.L. d'Alembert and J.L. Lagrange, stressed the principle of virtual velocities [Poisson 1842 (1833), I, pp.58-71; Grattan-Guinness 1990b, pp.318,325-7]. Jevons referred to Poisson's text in the chemistry Dictionary and in TPE when discussing the lever [Jevons 1863, p.489;

²³ Jevons had also attended De Morgan's classes before he left for Australia. See the discussion of Jevons' mathematics with regard to De Morgan in Grattan-Guinness 2002; Black 1972.

1970, p.147]. He could also have read a discussion of virtual velocities in the chapter "Application to Mechanics" in De Morgan's Differential and Integral Calculus. Like Poisson, De Morgan discussed the principle by referring to d'Alembert, arguing that it should be treated as an "axiomatic truth" [De Morgan 1842b, p.501; see also De Morgan 1842a, p.478].

The significance of the principle of virtual velocities in the variational mechanics tradition was the (problematic) claim that, with d'Alembert's action-reaction principle, all dynamic problems could be analysed in terms of statics and hence in terms of equilibrium.²⁴ As Poisson noted, the principle "implies ... that there is constantly an equilibrium between the given forces" [Poisson 1842 (1833), II, p.3]. De Morgan made the same point when referring to "D'Alembert's principle": "every problem of motion, of which the circumstances are known, may be reduced ... to one of equilibrium: that is to say, the properties of the actual variations which do take place may be investigated by means of the simple changes of place, without reference to time, which might be made in a system of rest" [De Morgan 1842b, p.510].²⁵ Jevons used the principle to argue that, because the lever displacements depicted "imaginary infinitesimal quantities", there was no requirement to integrate his basic equilibrium condition ("there is no effect to be summed up" [Jevons 1970, p.147]). The unstated import of the reference to the principle, however, was that it provided a rationale for treating all transactions as equilibrium outcomes in a market period. This was consistent with the way he referred to Mill's equilibrium tautology in the same chapter. In both cases, Jevons failed to explain the significance of his argument where marginal trades in the basic equilibrium condition (dy/dx) were virtual transactions that only existed in an imaginary form.

If Jevons did not clearly explain the significance of virtual velocities in the second edition of TPE, he had also buried a reference to the principle in the defence of his basic equilibrium condition in the first edition. As was noted above, he defended that condition by referring to a pendulum:

²⁴ Smith and Wise 1989, pp.372-77; Wise 1989a, pp.289-91; Grattan-Guinness 1990a, pp.276-83. See also the discussion of the significance of virtual velocities as a general balancing principle in Poisson 1842 (1833), I, pp.531-65.

²⁵ The use of virtual velocities entailed that "an unfortunate skier, tumbling down a slope with legs, arms, skis, and poles flying, would now have to be seen as a system in equilibrium" [Wise 1993, p.236].

It is much more easy to determine the point at which a pendulum will come to rest than to calculate the velocity at which it will move when displaced from that point of rest. Just so, it is a far more easy task to lay down the conditions under which trade is completed and interchange ceases, than to attempt to ascertain at what rate trade will go on when equilibrium is not attained [Jevons 1970, p.138].

Jevons had used the pendulum metaphor in Chapter I when justifying his treatment of behaviour as a balancing machine. He referred to De Morgan's Formal Logic (1847), which argued that the human mind could be represented as "an apparatus or piece of mechanism". A complex mechanism could be understood in different ways. Presented with a watch, for example, "One might use the pendulum and the weight, another the springs and the balance: one might discover the combination of toothed wheels, another a more complicated action of lever upon lever" [De Morgan 1926 (1847), p.30]. Similarly, the mind could be represented in different ways because "We have manifestations only, without the smallest power of reference to other similar things, or the least knowledge of structure or process, other than what may be derived from those manifestations. It is the problem of the watch to those who have never seen any mechanism at all" [ibid. pp.29-30]. Jevons used a similar argument in TPE when, citing De Morgan on the use of the balance "to detect equality and inequality" [ibid. p.202; Jevons 1970, p.82],²⁶ he argued that

It is from the quantitative effects of the feelings that we must estimate their comparative amounts. We can no more know or measure gravity in its own nature than we can measure a feeling; but, just as we measure gravity by its effects in the motion of a pendulum, so we may estimate the equality or inequality of feelings by the decisions of the human mind. The will is our pendulum, and its oscillations are minutely registered in the price lists of the markets [Jevons 1970, pp.83-4].

²⁶ As published in Jevons 1970, p.82, the paragraph immediately following the quotation from De Morgan appears to have been written by Jevons. It is actually a further quotation from De Morgan, as was clear in the first edition of TPE [Jevons 1871, p.11]. The mistake initially appeared in the second edition [Jevons 1879, p.10].

Having used the pendulum to justify the representation of actions as gravitational forces, Jevons indicated in Chapter IV that the metaphor was problematic. The point had also been acknowledged in the chemistry Dictionary when considering a balance "in the character of a compound pendulum", so as to analyse motion and hence its velocity. Jevons argued that, "Fully to understand the motions of a beam, it would be necessary to determine its moment of inertia round the axis, which is the sum of the moments of each particle, the moment of inertia being the movements of the particle multiplied by the square of its distance from the axis" [Jevons 1863, p.488]. He went on to note that "The mechanical problem of the balance is not so simple as may at first sight appear, and has not, so far as we are aware, been properly considered dynamically. The problem of the compound pendulum, will be found best treated by Poisson (Traite de Mechanique [sic], t.ii. c.i § 3)" [ibid. p.489]. Again, there was no explanation of the problem. The citation of Poisson's Treatise indicates, however, that Jevons was referring to the problem of friction which, Poisson argued, could be dealt with theoretically by the use of d'Alembert's principle and virtual velocities [Poisson 1842 (1833), II, pp.4-5]. That seems to be the import of Jevons' statement, when discussing the lever in TPE, that the "theory is only true for infinitely small displacements, and no sooner has the lever begun to move through any finite arc AA', than it ceases to be exactly true that the work done by P equals $P \times \text{arc AA}'$ " [Jevons 1970, p.145]. Justifying his use of virtual velocities, Jevons cited Poisson's statement that "the method of infinitely small quantities" had been "exclusively adopted" in his Treatise [Poisson 1842 (1833), I, p.11; Jevons 1970, p.147].

The problem of accounting for friction was also important for a number of experimental results reported in Poisson's Treatise. Poisson noted that, experimentally, it was only in the case of very small oscillations of a pendulum that the force of resistance was proportional to the square of the velocity. In effect, there was a marked difference between the gravitational constant "assigned by the calculus" and that obtained from the experimental results [Poisson 1842 (1833), I, p.285; II, pp.iv,31-2]. Jevons referred to that matter in an article that appeared in Nature in October 1871, just before the publication of TPE. Criticising a summary of non-Euclidean geometry by Hermann von Helmholtz,²⁷ he wrote that

The whole science of mechanics rests upon the motion of a uniform force, but where can we find such a force in operation? Gravity, doubtless, presents the

²⁷ See Richards 1988, ch.2.

nearest approximation to it; but if we let a body fall through a single foot, we know that the force varies even in that small space, and a strictly correct notion of a uniform force is only got by receding to infinitesimals ... [A] science of mechanics ... is generally only true of infinitesimals [Jevons 1871b, p.481].²⁸

Here Jevons was following Poisson and De Morgan, who had defined "Force" as "a name given to that which causes a change in the velocity of a body" and a uniform force as one where the velocity "acquired ... in any one interval of time is the same as that acquired in any other interval of equal duration" [De Morgan 1842a, p.29].

In the discussion of statics and dynamics in the first edition of TPE and the explanation of the lever added in the second, Jevons' defence of his basic equilibrium exchange condition was dominated by the question of whether it required integration. The significance of the principle of virtual velocities was presented only in that context. The integration question was, however, something of a side-show. The unstated role of the principle was that it provided a means to sidestep his cardinal difficulty in that it provided a justification for treating all transactions as equilibrium outcomes. It was symptomatic of Jevons' references to the attainment problem, however, that the full implications of his reference to virtual velocities were not clearly explained.

5. Thornton's Revenge

Jevons' references to his difficulty read like an elaborate game with the reader of TPE. On the one hand, he remarked that he had alluded to the problem and presented a series of clues to identify it. On the other hand, he did not clearly identify the problem and gave the impression that disequilibrium trades posed no difficulty for his analysis. A possible explanation for that rhetorical strategy is that, convinced he had found the truth, Jevons simply refused to clearly discuss any substantive difficulties he had identified with the theory. While this point is relevant, it would, if taken alone, reduce the explanation to a particular Jevons character trait.

²⁸ Jevons added, in a phrase he also used in TPE regarding the utility theory, that "plane geometry [is] only approximately true in fact, and exactly true in theory" [Jevons 1871b, p.481].

Consideration of the context in which TPE was written, however, provides a broader perspective to explain Jevons' peculiar treatment of the attainment problem.

Although it is not clear when Jevons recognised the problem, the available textual evidence suggests that it occurred between 1862, when the "Brief Account" of his theory was written, and late 1870, when he began working on the text of TPE. In the Brief Account, he referred to the properties of his equilibrium exchange condition with "infinitely small quantities of commodities":

The ratio of the increments of the commodities ... would be indeterminate but for the existence of a law that all quantities of the same commodity, being uniform in kind, must be exchanged at the same rate. The last increments, then, must be exchanged, in the ratio of the whole quantities exchanged. To explain in ordinary words how the adjustment takes place under this condition is almost impossible [Jevons 1866b (1862), p.284].

While that statement might be taken as an indicator of the attainment problem, it seems more likely that it simply referred to the difficulty in explaining the adjustment without the diagram used in TPE (Figure 1 above). That reading is supported by Jevons' preceding comment that "Whether the exchange will take place or not can only be ascertained by estimating the utility of the objects on either side, which is done by integrating the appropriate functions as limits. A balance of utility on both sides will lead to an exchange" [ibid. p.284]. A subsequent remark also indicates that, in 1862, Jevons saw no difficulty in referring to integration.²⁹ It should also be noted that a number of arguments in TPE required a utility integral. The discussion of the gains from international trade in Chapter IV, for example, was conducted in terms of the total utility of a nation [Jevons 1970, pp.172-4].³⁰ The diagrams used to illustrate the argument appear to date from the early 1860s.³¹ The adjustment-to-equilibrium

²⁹ "[T]hough the exchanges be regulated by equations, there cannot be equality in the whole utilities gained and lost, which are found by integrating the functions of utility of the respective commodities before and after exchange" [Jevons 1866b (1862), p.285].

³⁰ See also Figure 7 in Jevons 1970, p.160. That diagram was particularly problematic for Jevons as it referred to an indivisible commodity (see White 2001).

³¹ This is indicated by some notes in the Jevons Archive [John Rylands University Library of Manchester, JA 6/23/68-70] where the two separate aggregate utility diagrams in TPE appear on a single diagram. The handwriting suggests that the notes were written in the early to mid-1860s.

diagram, which probably dates from the same period [White 1989, p.486], also requires an integral if finite quantities were referred to, as in TPE. Taken together, these references suggest that, when he initially formulated the theory, Jevons saw no difficulty in referring to a total integral of utility.

In TPE, however, Jevons claimed that his approach was built on "the fearless consideration of infinitely small quantities" [Jevons 1970, p.78] and that, although

the mind often hesitates and is perplexed in making a choice of great importance ... [the theory] seldom involves the comparison of quantities differing much in amount. The theory turns upon those critical points where pleasures are nearly, if not quite, equal. I never attempt to estimate the whole pleasure gained by purchasing a commodity [ibid. pp.84,85].

If that statement reflected Jevons' defence of his basic equilibrium exchange condition and hence the attainment problem, the last sentence was clearly incorrect (see above). While this provided another source of confusion in understanding Jevons' argument, it indicates that he was unable to make all aspects of his previous analysis consistent with the stress on infinitesimals in TPE.³²

The attainment problem associated with disequilibrium trades had particular relevance in the context in which Jevons came to write TPE. As was noted above, in 1869 William Thornton's On Labour had attacked the relevance of references to the laws of supply and demand for explaining price formation. Focussing on trading during a market day (within a market period), Thornton defined an equilibrium as the trading cease-point that occurred when all commodity stocks were sold or a sellers' reservation price was reached. He argued that, if demand and supply were defined as the prices at which the purchasers and sellers were prepared, respectively, to trade different quantities, references to demand and supply could, in themselves,

³² It has been argued that, following De Morgan, Jevons' use of the calculus was something of a blancmange of references to a theory of limits combined with ratios and differential coefficients ("those queer brutes that are smaller than ordinary quantities but not zero" [Grattan-Guinness 2002, p.693]). Grattan-Guinness suggests that Jevons "used the calculus best" in the Brief Account, as "clarity was reduced" in the second edition of TPE with the citation of Poisson on the use of infinitesimals [ibid. p.694]. If clarity was reduced, I suggest it was driven by the attainment problem.

explain little about how the equilibrium position was reached. The process of price adjustment required an explanation of market coordination that depended on the interdependent behaviour of heterogeneous sellers with different access to credit facilities, experience and, therefore, knowledge. The signal for price adjustment during the day was the existence of unsold stocks, which meant that most trades occurred at disequilibrium ("intermediate") positions [Thornton 1869, pp.43-77].³³

If Thornton had introduced the question of how an equilibrium position was to be explained by coordination in a market, his focus on disequilibrium trading created a substantive difficulty for those who then attempted to restore the laws of supply and demand to the principal explanatory role in a market period. While dismissing Thornton's critique, in the first set of (quantity-function) supply and demand diagrams published in English, the engineer H.C. Fleeming Jenkin was confronted with an equilibrium attainment problem. Although the precise rationale for the shapes of his diagrams was unclear, Jenkin acknowledged that the demand curve for trading on a market day required a given set of expectations as a parameter. Disequilibrium trades would change the expectations and shift the function [Jenkin 1870, pp.153,154,155]. Jenkin then asserted that, with a uniform price set at any moment by market specialists, all trades during the day took place at equilibrium ("at each moment the ... law of supply and demand holds good" [ibid. p.156]).

Jevons subsequently acknowledged that he had written up TPE for publication in part because Jenkin's article made no mention of Jevons' utility theory that had been discussed in the exchange of letters between them in 1868 [Jevons 1911, p.lvii; Black 1973-81, iii, pp.166-78]. The terms of the debate over Thornton's critique also left their mark on Jevons' discussion of market-period trading in Chapter IV of TPE. In part this was evident from a number of references to Thornton [Jevons 1970, pp.148-9,158-9] and the focus on trading within a day during a market period. Less obvious were two effects of Jenkin's analysis. First, the engineer's discussion of indeterminacy in exchange with an indivisible commodity appears to have prompted Jevons' reassessment of his own analysis of that topic [White 2001]. Second, in TPE, Jevons adopted Jenkin's tactic of claiming that market specialists (brokers) set a

³³ See White 1994b, pp.151-7. My earlier account is, however, marred by the suggestions that Thornton was using a "path-dependent adjustment process" to equilibrium and that he regarded the equilibrium position as "virtually irrelevant" [ibid. pp.154,157].

uniform price at any moment. In both cases, the tactic was used to sidestep an attainment problem that followed the attempt to depict or explain the laws of supply and demand in a functional form. The manifestation of Jevons' problem was different from Jenkin's because he did not use supply and demand functions. Nevertheless, it had the same basis in disequilibrium trades shifting a parameter. Jevons' initial failure to identify the problem, his inconsistent arguments in TPE regarding references to total utility and his use of market specialists to set a uniform price at any moment, might suggest that Jevons identified his particular indeterminacy problem after reading Jenkin's article. The inconsistencies noted above in TPE regarding references to total utility could thus reflect the way in which Jevons hurriedly wrote up TPE to establish his own contribution to the debate over the laws of supply and demand.

Whether Jevons recognised his attainment problem before or after reading Jenkin's article, it was a serious liability in the wake of Thornton's critique. In TPE, the most positive comment that Jevons felt able to make about On Labour was the following:

Though Mr Thornton's objections [to explanations of price formation in terms of the laws of supply and demand] are mostly beside the question, his remarks have served to show that the action of the laws of supply and demand was inadequately explained by previous economists. What constitutes the demand and the supply was not carefully enough investigated [Jevons 1970, p.149].

While Jevons attributed a positive, albeit limited, role to On Labour, it was extraordinary to claim that Thornton's analysis was "mostly beside the question". Thornton had argued that references to the laws of supply and demand could not provide a substantive explanation for the adjustment to equilibrium in a market period, and that was precisely the problem that Jevons could not solve within his own framework. It was that context that seems to explain, in large part, his rhetorical strategy in TPE of alluding to, but refusing to explain, the cardinal difficulty of the whole theory.

Conclusions

When Jevons delivered a defence of "the" mathematical theory of political economy to the Manchester Statistical Society in 1874, he drew attention to the fundamental

similarity between his basic equilibrium condition of exchange in Chapter IV in TPE and that given in the first volume of Walras' Elements (1874). Jevons argued that

M. Walras and myself have, in entire independence of each other, by paths of reasoning which are, in the forms of expression, the methods of illustration, and all merely incidental circumstances, as different as they well could be, yet arrived at identical results. This fundamental formula of the science of economy is far from being of an obvious character, so that the coincidence cannot possibly be attributed to chance. The only other explanation which can be entertained is that it arises from the sound and truthful investigation of the subject [Jevons 1875, p.3].

While the fundamental formula for equilibrium in a market period was far from being of an obvious character, it did not necessarily follow that the similarity in results was due to Jevons and Walras having arrived at 'the truth'. For there was another possible explanation - they had both used the metaphor of a lever in static equilibrium to derive their formula. If Jevons did not enlighten his audience regarding the metaphor in 1874, he reiterated the claim made in TPE that "the laws of supply and demand, as generally accepted by economists, are easily derived from the theory of exchange, so that the theory is verified by experience and statistical science" [Jevons 1875, p.15]. This was ingenuous. As Jevons realised, the cardinal difficulty of the whole theory was that the mechanical metaphor of adjustment to equilibrium with a given exchange ratio was incompatible with the changing exchange ratio (price) required by the laws.

If the problem of the attainment of equilibrium has continued to dog post-classical theory [Fisher 1987],³⁴ it is odd that the secondary literature on TPE has devoted little attention to identifying Jevons' difficulty and hence to explaining why he only alluded to, rather than clearly explaining, the problem. In part, this is because the problem was simply ignored. Knut Wicksell, for example, while using Jevons' crossed and superimposed utility diagrams to represent the transactions of a representative actor, simply assumed that "on the market one price or a proportion of exchange between every two commodities establishes itself within a short time for each commodity in

³⁴ See also Trevor Swan's reference to "'virtual' displacements" (characterised, with a further Dodgsonian metaphor, as "a grin without a cat") in post-classical growth theory [Swan 1956, p.351] and the apposite commentary in Harcourt 1972, pp.34-9.

which afterwards the bulk of transactions are done" [Wicksell 1954 (1893), pp.58-9,70-1].

A second strand in the literature read Jevons' distinction between statics and dynamics to make it consistent with later, quite different, arguments. This approach first appeared in the exchange between Francis Y. Edgeworth and Ladislaus von Bortkiewicz regarding the validity of Walras' tatonnement explanation for equilibrium. Noting Jevons' use of virtual velocities, Edgeworth insisted that the account in TPE was restricted to a "static" depiction of an equilibrium position and that Jevons had used the term "dynamics" to refer to the adjustment process by which an equilibrium was attained [Edgeworth 1889, p.435; 1891, pp.13-14]. Bortkiewicz, writing under Walras' "close supervision" [Bridel and Huck 2002, p.527], noted that Jevons had used the term statics when referring to transactors exchanging "until they come to equilibrium". This was, he argued, the same process that had now been explained by Walras' tatonnement [Bortkiewicz 1891, pp.85-6]. Edgeworth, however, simply refused to discuss the quotation to which Bortkiewicz had drawn attention. While Bortkiewicz and Walras were strictly correct, where Edgeworth was not, in noting how Jevons had used the term statics, the acrimonious exchange failed to explain Jevons' argument. Further confusion followed nearly a century later in an account which, in replicating Edgeworth's reading of the meaning of statics and dynamics in TPE³⁵ and referring to Jevons' defence of differential coefficients, suggested that "[b]ehind this 'statical view of the question' there must be a dynamic process of trading. What Jevons had in mind is a piecemeal exchange process" of recontracting and arbitrage [Negishi 1982, pp.222, 226; see also Creedy 1986, pp.44-5; Dome 1994, pp.95-7]. TPE's exchange theory is then represented by an Edgeworth-Bowley box where recontracting explains the "law of indifference ... that plays the role of the equilibrium condition for Jevons ... Demand equals supply trivially, since the quantity of a commodity given by person A is equal to the quantity received by another person B" [Negishi 1994, p.xx]. Putting to one side that Jevons did not refer to recontracting, which is inconsistent with his discussion of market-day trading,³⁶ this misunderstands his distinction between statics and dynamics. In TPE, dynamics referred to the relation between a long period and a series of market periods. Statics included an explanation of an

³⁵ Bridel and Huck 2002, pp.525-32, also follow Edgeworth's characterisation of statics and dynamics in TPE.

³⁶ See, for example, Jevons' discussion of the New York Stock Exchange, cited above.

adjustment to equilibrium (although Jevons was unable to do so). Any attempt to represent Jevons' analysis in terms of an Edgeworth-Bowley box (see also Creedy 1992, pp.128-39) thus erases his particular distinction between statics and dynamics as well as his depiction of equilibrium prevailing at any moment in a market period.

The importance of paying close attention to Jevons' language is not simply to locate the reference points for, and hence the meaning of, particular categories. It is also essential for unravelling the ways in which he attempted to deal with his cardinal difficulty. Although Jevons explicitly used the tautological definition of equilibrium in his primer (was it a tale for children?), the use of momentary equilibria in TPE seems to have been an attempt to evade the attainment problem, rather than a belief that disequilibrium transactions were irrelevant in "actual" markets. In noting Jevons' references to actual markets, a third strand of commentary has suggested he drew a clear (and apparently unproblematic) distinction between the theoretical analysis of equilibrium and a real world of disequilibrium trading [Ekelund and Thommesen 1989, pp. 583-584; Schabas 1990, p.90; Peart 1998]. Those accounts do not, however, satisfactorily explain why Jevons insisted that the theoretical conception of a perfect market is more or less completely carried out in practice. Nor do they clarify the analytical roles of brokers and the principle of virtual velocities in his analysis. The problem here, at least in part, seems to be an implicit assumption when reading TPE that Jevons provided a transparent story of market trading, devoid of rhetorical guile. To this extent, Jevons' rhetorical strategy of only alluding to his equilibrium attainment problem, so as to obscure its presence and significance, was successful.

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FIGURE 1

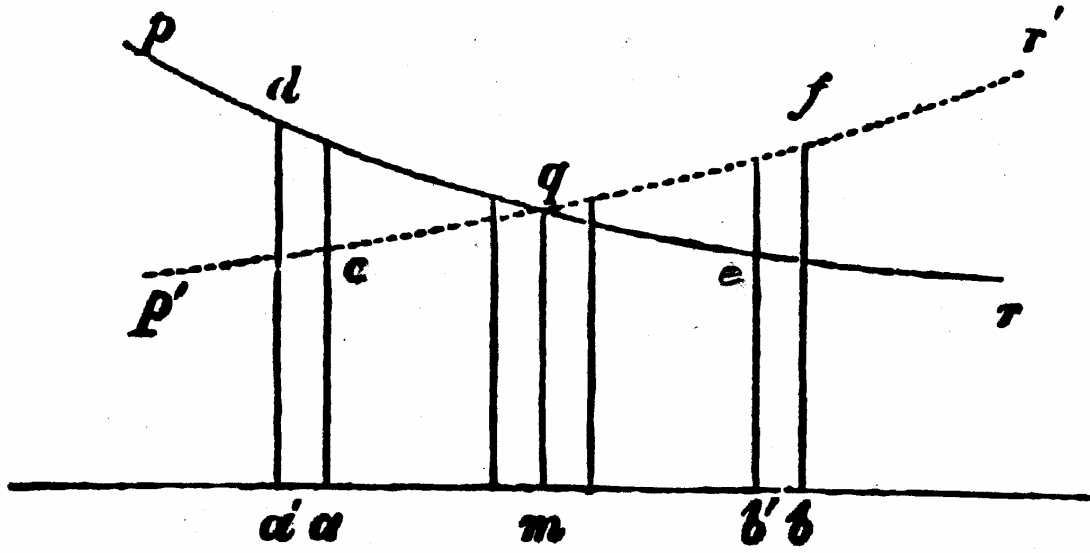


FIGURE 2

