Comments on Lewis and MacDonald (2004)

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February 2005
Revised June 2006[1]

This note arises from an exchange between Dowrick and Wells (DW 2004) and Lewis and Macdonald (LM 2002, LM 2004) recently published in the *Economic Record* in relation to the formulation and interpretation of expressions for the demand for labour at the aggregate level. It is our contention that LM 2004 continues to promote erroneous arguments.

1. The LM assumption that the price level is exogenous

The assumption of an exogenous price level for aggregate output is central to the LM 2004 Reply. They state:

"In common with the standard (neo) classical model, Lewis and MacDonald (2002) implicitly assume that wages are determined by supply and demand and that the price level is exogenous." LM (2004, page 442, first para.)

An exogenous price level is, however, inconsistent with the neo-classical model at any level of analysis other than that of the individual firm.[2] To see this, consider the assumption that the output price is exogenous while the nominal wage is influenced by changes in supply conditions. This assumption is incompatible with neo-classical competitive equilibrium where production exhibits constant returns to scale. Profit maximisation implies that price equals marginal cost, and constant returns to scale implies that marginal cost equals average or unit cost. A rise in the cost of a factor must raise unit costs and must therefore raise the equilibrium market price. The assumption of an exogenous market price which does not respond to changes in nominal factor prices involves stepping outside the neo-classical model of market equilibrium.

Putting this point another way, consider the impact of a wage rise in a neo-classical economy in competitive equilibrium. (This should be understood in the context of standard comparative static analysis – we are not analysing changes between one time period and the next, over which time technological progress might have occurred, but with hypothetical marginal changes at one point in time.) With constant returns to scale and common technology, competition ensures that all firms are making zero profits. If the market price does not rise in response to the wage rise, firms will make strictly negative profits at any strictly positive output and the profit-maximising response is to reduce output and factor inputs to zero.

The only situation in which the LM assumption of an exogenous price level makes sense is that of an individual, vanishingly-small firm with sunk capital costs facing a

[1] Thanks to Robert Dixon for pointing out an error in a previous version of this note.
[2] The assumption of an exogenous price level is also inconsistent with the behaviour of the Australian macroeconomy over LM's (2002) estimation period, 1961-98. Inflation targeting was introduced only in the 1990s, and the targeting is explicitly inexact. The notion that the price level was exogenous during the 1970s is particularly hard to credit.

firm-specific wage rise, where it treats the competitive market price as exogenous because it is too small to influence the market. In this situation, the firm will respond with a finite reduction in output and employment (rather than reducing both to zero) only in the situation where investment is irreversible, i.e. if capital is a fixed factor, and the firm is seeking to maximise short-run net revenue. This situation, however, does not conform with the long-run competitive equilibrium that LM (2002) claim to analyse.

In other words, it is not the case that our results (DW 2004) differ from LM (2002) because they were making an alternative (albeit unstated) but equally plausible assumption that the price level is exogenous, but because their assumption, made explicit in LM (2004), is inconsistent with neo-classical market equilibrium.

The assumption of an exogenous price is also inconsistent with their discussion of the scale effect on labour demand. The endogeneity of the competitive price is implicit in LM's (2002, p.21, para.3) discussion of the "scale effect arising from output expanding as costs of production fall". They go on to explain that the scale term, $s\eta$, "arises from increased output due to a fall in costs of production." In their reply, LM (2004, p443, point 3) state that in response to a fall in real wages "there is a rise in labour demand as firms now find activities profitable which were previously unprofitable at the higher real wage." Why should the quantity traded in a competitive market rise in response to falling costs? The explanation is that the output price falls by s (i.e. it is endogenous with respect to factor prices) and demand rises by $s\eta$ where s is the share of labour in revenue and $-\eta$ is the own-price elasticity of demand for output.

This is exactly how Hamermesh (1993, p24) defines the terms s and $-\eta$, and he states (also p. 24): "When the wage rate increases, the cost of producing a given output rises. In a competitive product market a 1 percent rise in a factor price raises cost, and eventually product price, by that factor's share." The endogeneity of the price of output in market equilibrium is further clarified by Hamermesh's (1993, p. 27) statement in deriving the "fundamental law of factor demand" under constant returns that "Under competition firms equate price, p, to marginal and average cost".

Their failure to accept that the competitive market price is a function of factor prices leads LM (2004, p.442, para.2) to say that "The Dowrick and Wells results … follow from *their alternative, arbitrary, assumption* that when there is a fall in wages the price level changes by exactly the amount of the fall in unit labour costs." (emphasis added) LM (2004) proceed in the same paragraph to further ridicule our 'arbitrary' assumption.

Our assumption is far from arbitrary. It corresponds exactly to the neo-classical analysis of the competitive market price under constant returns production technology, as in Allen's (1938) analysis and the unambiguous quotes reproduced above from Hamermesh (1993). Moreover, as we demonstrate in the following section, the elasticity formulae that are asserted in LM (2002) and re-asserted in LM (2004) are still not the correct formulations for the response of labour demand with respect to changes in the real wage.

2. Is the labour demand elasticity defined with respect to the real wage or the nominal wage?

LM (2004, pp. 442) argue that their assumption of an exogenous price justifies the claim in LM (2002, p.21) that the expression for the output-constant elasticity of labour demand, $-(1-s)\sigma$, is defined with respect to the real wage. LM (2004, p.443, para 3) claim that "all derivatives were in fact with respect to the real wage and real cost of capital". To justify this they refer to Hamermesh (1993) without any page number and to "a range of studies of the demand for labour too long to list here".

However, even if we work with the assumption that the price level is exogenous, the LM (2002) claim is still incorrect. To demonstrate this, we restate equations (7) and (8) from DW (2004), equations which LM (2004) also reproduce:

$$\ln L\left[\boldsymbol{W}, Q, A\right] = \sigma \ln \alpha - \sigma\left[\ln W - \ln P(\boldsymbol{W}, A)\right] + \ln Q[P(\boldsymbol{W}, A)] - \left(1-\sigma\right)\ln A \quad \text{(DW7)}$$

$$\begin{aligned} \frac{d \ln L[\boldsymbol{W}, Q, A]}{d \ln W} &= -\sigma\left[1 - \frac{\partial \ln P(\boldsymbol{W}, A)]}{\partial \ln W}\right] + \frac{\partial \ln L[.]}{\partial \ln Q}\frac{\partial \ln Q}{\partial \ln P}\frac{\partial \ln P(\boldsymbol{W}, A)}{\partial \ln W} \\ &= -\sigma(1-s) - \eta s \end{aligned} \quad \text{(DW8)}$$

where $\boldsymbol{W}$ represents the vector of factor prices, (W, R), P is the price of output and $Q(P)$ is the demand for output.

If we allow the assumption of an exogenous price level, $\bar{P}$, i.e. setting the terms $\partial \ln P$ *and* $d \ln P$ in (DW8) to zero, and conditioning on the level of output, $d \ln Q = 0$, we obtain the partial derivative of $\ln L$ with respect to $\ln W$, which now represents the output-constant labour demand elasticity with respect to the real wage:

$$\frac{\partial \ln L\left[\boldsymbol{W}, Q, A\right]}{\partial \ln(W / \bar{P})} = \left.\frac{\partial \ln L\left[\boldsymbol{W}, Q, A\right]}{\partial \ln W}\right|_{\bar{P}} = -\sigma$$

This result, allowing the assumption of an exogenous price level, again contradicts the claims of LM (2002 & 2004). The output-constant elasticity of labour demand with respect to the real wage is equal to the elasticity of factor substitution. We can only derive the LM (2002) and Hamermesh (1993) result that the output constant elasticity of labour demand equals $-(1-s)\sigma$ if we allow the output price to be a function of factor prices (as Hamermesh does – see quotes above) and differentiate with respect to the nominal wage.

Alternatively, taking LM's (2002) own equation (5), substituting their expression for A and noting that the bracketed ratio on the right-hand-side of the equation should be (w/p) rather than (w/ρ):

$$\ln L = \sigma \ln \alpha - \sigma \ln\left(w/p\right) + \ln Q \quad \text{(LM5)}$$

Differentiating with respect to the real wage, w/p, whether or not we treat p as exogenous, we find again that the output constant elasticity of labour demand with respect to the real wage is equal to σ, not $-(1-s)\sigma$:

$$\frac{\partial \ln L[(w/p),Q]}{\partial \ln(w/p)} = -\sigma$$

In fact, this result is recognised by LM (2004, p.442, top of right-hand column) where they restate "the result .. that the derivative of the marginal productivity condition with respect to the real wage, the elasticity of substitution, is σ."

It is at this point that we see why LM (2002) are at pains to deny that expressions such as their own equations (5) and (6) (or DW7, reproduced above) represent the demand for labour. They say on pages 20-21:

"It is important to note that this is not a demand for labour curve but a marginal productivity condition. It is wrong to mistake this for a demand for labour curve and hence interpret the coefficients as elasticities of demand with respect to real wages and output. .. σ .. is the *elasticity of substitution* between labour and capital *not*, as stated in most of the literature, the output constant elasticity of demand for labour."

We have to again contradict LM (2002). The function $L(\boldsymbol{W},Q)$ in the (re-arranged) marginal productivity condition is precisely the conditional demand for labour that arises when a firm maximises profit / minimises cost conditional on the level of output and on factor prices. This is precisely what Hamermesh (1993, p.30) calls "the demand for labor" and what Varian (1978, p.32) refers to as "the firm's conditional factor demand". Moreover, "most of the literature" is correct if it is equating the elasticity of substitution, σ, to the output-constant elasticity of demand for labour with respect to the real wage.

Some of the confusion over the definition of the elasticity of factor demand may have arisen from a misunderstanding of Hamermesh's (1993) analysis. On page 24, Hamermesh states the own-price conditional elasticity of demand for labour as $\eta_{LL} = -(1-s)\sigma$ without specifying whether the elasticity is defined with respect to the real or nominal wage. He does not derive the result explicitly, but tells us that he is following Allen (1938). In Allen's analysis (section 14.8, "The demand for factors of production") the assumptions and derivation are explicit. Allen treats factor prices as given and the competitive market price, p, as an endogenous variable which is equal (in the case of CRS production) to the constant average cost. He then derives, on page 373, the expression $-(1-s)\sigma$ for the factor demand elasticity in relation to a variation in the nominal factor price, p_a, not in relation to the factor price / product price ratio, p_a/p.

3. The LM assumption that the elasticity of demand for output equals –1

LM (2004, p.443, final paragraph) state that "At the firm level, the interpretation is simply the elasticity of demand for its output". Whereas in DW (2004) we argue that this interpretation carries through at the macroeconomic level to the own-price elasticity of demand for final output (i.e. the elasticity of the aggregate demand curve), LM (2004, p. 4443-4) suggest that "the value of η depends on assumptions regarding both the demand for and the supply of goods." They proceed to a theoretical argument for the proposition that the elasticity of demand for output equals –1. They state the following:

"However, if the natural rate, and hence effective labour supply, changes over time the real wage rate will change and the economy will move down (or up) the aggregate demand for labour curve; employment and output will change simultaneously. Since the economy must be in equilibrium the value of the change in output produced must equal the change in value of factors of production. For an economy exhibiting constant returns to scale it only makes sense for a one percent fall in real factor prices to be accompanied by a one per cent rise in output and factor demand. That is, η is equal to –1."

The first sentence is broadly correct, although it would be more accurate to say that the economy moves up or down the aggregate demand curve for output, and that the labour market moves up or down the aggregate demand curve for labour (labour demand being a derived demand). The second sentence is true by definition: the value of GDP is equal to the sum of factor payments.

However the argument in the third sentence, relying on the appeal that "it only makes sense", is incorrect. The response of aggregate demand to a change in the price of output is an empirical question. Were it the case that "a one percent fall in real factor prices [to] be accompanied by a one per cent rise in output and factor demand", the value of factor payments and GDP would be unchanged in response to the predicated shift in labour supply – a highly unlikely outcome in any neo-classical model of the aggregate economy. Moreover, if we were to add the LM assumption that the price of output is fixed, this would imply that the level of real output would also be unchanged – in contradiction to their statement in the first sentence quoted above.

Hence, the conclusion "η is equal to –1" is analytically unjustified. The elasticity of aggregate demand with respect to the price of output is, in the neo-classical model, ultimately determined by the preferences of domestic and foreign households. In practice, η is a key parameter to be determined econometrically. It might turn out to be equal to –1 in some cases, but there is no general theoretical case that it should be so.

4. The LM treatment of Hicks-neutral (factor-neutral) technical change

LM (2002, p.20) specify their production function (2) without time-varying technology. When it comes to specifying their estimating equation (6), they add to equation (5) "a time trend to allow for Hicks-neutral technical progress", yielding:

$$\ln L_t = \alpha_0 + \alpha_1 \ln\left(\frac{W}{P}\right)_t + \alpha_2 \ln Q_t + \alpha_3 t + u_t \qquad \text{(LM6)}$$

Hicks-neutral technological progress is equally labour-saving and capital-saving. In DW (2004) we explicitly augment the LM (2002) production function with Hicks-neutral technology and derive the result that the coefficient α_3 in (LM6) is equal to $-(1-\sigma)\lambda$, where λ is the rate of Hicks-neutral technological change and σ $(=-\alpha_1)$ is the elasticity of factor substitution. This implies that $\lambda = -\alpha_3/(1+\alpha_1)$. The LM (2002) quarterly employment regression has a point estimate of –0.46 for α_1 and –0.003 for α_3, implying that the annual rate of Hicks-neutral technological change is around 2.2 per cent per annum. We have suggested, and now repeat, that this is implausibly high for the time period under consideration, for which ABS estimates of total factor productivity growth average around one per cent per year. The ABS definition of TFP growth corresponds to Hicks-neutral technical progress.

Alternatively if LM were to treat technical progress as being labour saving (i.e. *Harrod*-neutral) then one should start with a production function reflecting that assumption. For example, Dungey and Pitchford (1998, p.220) adopt the specification

$$Q_t = \left[\alpha_t L_t^{\rho} + \beta_t K_t^{\rho}\right]^{h/\rho};\ \rho < 1,\ h > 0,\ \alpha_t = ae^{\lambda t},$$

which, assuming constant returns to scale yields

$$\ln L[W, P, Q, A] = \sigma \ln a + \sigma \ln\left(W/P\right) + \ln Q + \sigma\lambda t\ .$$

In this case, the transformation to recover the rate of technical change is $\lambda = \alpha_3/\sigma = \alpha_3/\alpha_1$. Given their negative estimated values of $\alpha_3 = -0.005$ and $\alpha_1 = -0.40$ (Dungey and Pitchord (1998, p.222), their results imply labour-saving technical change at a rate of 1.25% p.a.

Alternatively, one could interpret labour-saving technical change by specifying a constant-returns CES production function with a pure labour-augmenting term:

$$Q_t = \left[\alpha\left(A_t L_t\right)^{\rho} + (1-\alpha)K_t^{\rho}\right]^{1/\rho};\qquad \rho < 1,\qquad A_t = A_0 e^{\lambda t}$$

This case yields an expression of the form

$$\ln L[W, P, Q, A] = \sigma \ln \alpha - (1-\sigma)\ln A_0 - \sigma \ln\left(W/P\right) + \ln Q - (1-\sigma)\lambda t$$

which turns out to be identical to the conditional labour demand function in the case where technology is factor neutral.

We have not checked on the specific technical change assumptions for each of the six studies cited in Table 1 of the LM 2004, and so offer no opinion as to whether any of the studies are in fact comparable to LM 2002 in terms of the way technical change is incorporated in their assumed production technology. However it is clear that in at least two cases the estimates are not comparable – Dungey and Pitchford (1998) estimated a parameter on the time trend which is not directly comparable with the estimate reported in LM 2002. The same is true of Commonwealth Treasury (1996, p.1.2) which makes it clear that Harrod-neutral technical change is being assumed.

5. Other errors or inconsistencies in the LM reply

(i) LM (2004, p.441 and 442) have two errors in the translation of equation (2) – which is apparently taken from a mimeo version of DW (2004) without attribution – to equation (4). The first term on the right-hand-side of (4) should be $\sigma \ln \alpha$ not $\sigma \ln b$. The same error appears in equation (7).

(ii) LM (2004, p.442, equations 7-9). These equations have not been accurately reproduced from DW (2004). The term 'W' in the labour demand and price functions should be in bold, '$\boldsymbol{W}$', to represent the vector of factor prices.

(iii) LM (2004, p.4) state that 'All that is necessary for their validity and consistent estimation is that real wages are (weakly) exogenous in the marginal productivity condition'. ['their' in this quotation refers to the LM results].

We have not, in DW (2004), included comments on the LM econometric estimation procedures. However, it is crucial to ensure that what is being estimated can be identified as the labour demand curve (or as LM would have it, the marginal productivity condition) and not some amalgam of these and the labour supply curve. The phrase 'All that is necessary' in the above quotation could perhaps be extended and re-worded to make some reference to econometric identification.

(iv) LM (2004, p.443, 3rd para. In right-hand column) misquote us as saying that

$-\sigma(1-s)-\eta\sigma$ describes the "total elasticity of aggregate demand with respect to the nominal wage" (no page reference given).

If the reference is to our statement in the fourth paragraph of the right-hand column of Dowrick and Wells (2004, p.438) then what we actually say is that "We see from equation 8 that this formula is in fact the unconditional elasticity of aggregate demand for labour with respect to the *nominal* wage". Our wording has been changed in a number of ways, most significantly by omitting the words 'of labour'.

References

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