THE EFFICIENT RECREATIONAL USE OF A NATURAL RESOURCE

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ABSTRACT

This paper considers the efficient use of a natural resource in which increased recreational use results in increased degradation. Users treat degradation as an external cost, and do not take it into account in their choice of use. The social surplus is the product of quality of the natural resource (as measured relative to its pristine state) and the value of recreational use of the natural resource (in its pristine state). The surplus need not be concave and thus may not have a unique local maximum. The level of demand and relative variability of the elasticity of quality and elasticity of benefit are the determinant of the number of local maxima of the surplus. When quality is always less variable than that benefit there are a maximum of two local maxima of the surplus. The impact on the efficient level of use of demand increases are described.

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Use often degrades the quality of a natural resource. For example, increased use of walking tracks through wilderness can cause erosion along the track and of nearby areas, while increased use of lakes for fishing can reduce fish stocks. As demand for the use of natural resources increases, it must be decided whether to control degradation by regulating access. Popular opinion on the regulation of use of a natural resource often polarises. Some believe there should be no use of the natural resource, while others believe that there should be unfettered access for all.

To assess the implication of such views, this paper presents a model of the recreational use of a natural resource. Central to the model is a measure of the quality of the natural resource. Quality is defined to capture the principal features of the relationship between deterioration and the recreational use of the natural resource. Specifically, it is assumed that low levels of use may not cause deterioration. However beyond a threshold use level there is a region of deterioration, over which the quality of the natural resource declines. The natural resource reaches its minimum level of quality at the upper use level in the region of deterioration. Use beyond this level does not cause any further degradation.

Users treat degradation as an external cost, and do not take it into account in their choice of level of use. There is a role for a policy that offsets the impact of the externality in order to achieve the efficient level of use. This would be a straightforward economic problem if the society's objective function were concave. However, as noted by Starrett (1972) and Baumol and Oates (1988), such problems often exhibit non-concavities. In this event society's objective function may have multiple local maxima (ie. a number of use levels may satisfy the first order conditions for efficiency) and pricing may not be able to achieve the globally efficient level of use.

The model analysed in this paper is similar in approach to that developed by Fischer and Krutilla (1972). It is cast more formally so that the issue of non-concavity can be addressed. The social surplus is the product of quality (as measured relative to its pristine state) and the value of recreational use of the natural resource (in its pristine state). The quality function is not concave, and thus the surplus need not be concave. However it is demonstrated that pricing is 'usually' capable of achieving the efficient level of use. It is shown that the efficient use level is greater than zero. If demand is sufficiently low the surplus is concave, and therefore exhibits a unique local maximum. At higher demand levels the surplus need not be concave, and may not have a unique local maximum. Indeed there could be an arbitrarily large number of local maxima of the surplus.

It is shown that, in conjunction with the level of demand, the relative variability of the elasticity of quality and elasticity of benefit with use determines the maximum number of local maxima of the surplus. Three cases are considered in detail. In the first the variability of quality with use is greater than that of the benefit over the range of use levels in which quality declines. Such a condition is likely to hold if quality declines rapidly toward its minimum level once the threshold use level is exceeded. In this case it is demonstrated the surplus exhibits at most two local maxima at higher levels of demand: a 'low use' outcome in which price is used to restrict numbers so that quality is above its minimum level, and a 'high use' outcome in which use is unrestricted and quality falls to its minimum level. A criterion is given to determine which of the use levels is the globally efficient level.

The case in which the variability of quality with use is less than that of the benefit over the range of use levels in which quality declines is also considered. This condition is likely to hold if the natural resource is robust to use. The robustness with respect to use could exist because some infrastructure is in place. For example, walking tracks may have been hardened in some way to prevent deterioration. In this event it is demonstrated that there is a unique local maximum of the surplus. Finally the case is considered in which the variability of quality with use is initially greater than the variability of the benefit, but as use increases the variability of the quality becomes less than the variability of the benefit. This case is likely to hold if significant deterioration of the natural resource is causes by low levels of use, but following this initial deterioration the rate of deterioration becomes stable. In this event it is demonstrated that there is at most two local maxima of the surplus.

The model is used to analyse the impact of an increase in demand. It is argued that an increase in the demand per user does not affect the efficient level of use. However either an increase in the number of users or a loss of substitute activities changes the globally efficient level of use. An analysis of the impact of an increase in the number of users is conducted under the assumption that variability of quality with use is greater than that of the benefit. The 'high use' outcome is shown to become the efficient one if there is a sufficiently large number of users. However the surplus per user is shown to decline as the number of users increases.

Section 1 states the assumptions underlying the quality function. In section 2, the demand for the natural resource by individuals is modelled. The impact of pricing is also determined. The central results of the paper are derived in section 3. The social surplus is defined and is used to consider efficient use levels. In particular, the number of local maxima of the surplus is related to a measure of the relative variability of quality and user benefit. In section 4 the impact of an increase in demand per user and the impact of an increase in users is discussed. Section 5 concludes the paper.

1. Quality of the Natural resource

The natural resource has a level of quality, which is evaluated from the perspective of users. The quality of the natural resource varies with use q(N), where N is the number of uses per year of the natural resource. Assume that quality has been defined in such a way that the pristine state of the natural resource is assigned a quality of 1, i.e. $q(0) = 1$.

The natural resource is assumed robust at low levels of use, so increased use does not reduce quality. At these low use levels, increased use may make access to the natural resource easier (for example a track may form in a previously untouched area) and quality may increase. However beyond a threshold use level, $\underline{N} \geq 0$, quality declines with use. The rate of quality decline increases with use until it reaches a maximum at use level $\hat{N}$. At higher levels of use the rate of decline of quality slows, and quality plateaus at a degraded level once usage reaches $\bar{N}$. The set of use levels for which quality declines, $D=[\underline{N}, \bar{N}]$, is termed the region of deterioration

Assume that q(N) is differentiable, while $q'(N)$ and $q''(N)$ are continuous for all N and differentiable for all N except possibly $N= \underline{N}$ and $N=\bar{N}$. From the above discussion $q'(N) \geq 0$ for $0 \leq N \leq \underline{N}$, $q'(N)<0$ for $\underline{N}<N<\bar{N}$ and $q'(N)=0$ for $N>\bar{N}$. Further $q''(N)<0$ for $\underline{N}<N<\hat{N}$ and $q''(N)>0$ for $\hat{N}<N<\bar{N}$ and $q''(N) = 0$ for $N=\hat{N}$ and $N>\bar{N}$. A quality function satisfying these conditions is depicted in figure 1.

Define the elasticity of quality by $E_q(N) \equiv \dfrac{Nq'(N)}{q(N)}$. Under the above assumptions $E_q(N)$ is continuous for all $N \geq 0$. In addition $E_q(N) \geq 0$ for $N \in (0,\underline{N})$, $E_q(N)= 0$ for $N=\underline{N}$,

$E_q(N)<0$ for $N \in (\underline{N},\bar{N})$ and $E_q(N)=0$ for $N \geq \bar{N}$. $E_q(N)$ and $E_q'(N) = \frac{d\,E_q}{dN}$ are continuous for all N and differentiable for all N except possibly $\underline{N}$ and $\bar{N}$.

The above assumptions place some restrictions on $E_q'(N)$ on D. For example, $E_q'(N) < 0$ for all $N \in [\underline{N}, \hat{N}]$. However the sign of $E_q'(N)$ is ambiguous over $[\hat{N},\bar{N}]$. As $E_q'(\hat{N}) <0$ and $E_q'(\bar{N}) >0$ there must be at least one local minimum of $E_q(N)$ in $[\hat{N},\bar{N}]$. When graphing $E_q(N)$, as in figure 1, it is assumed that $E_q(N)$ has a unique minimum in D at the use level $\bar{\bar{N}}$. As $E_q'(\hat{N}) <0$, then $\bar{\bar{N}}>\hat{N}$. It is shown in appendix 1 that $E_q(N)$ has a unique minimum if $\bar{\bar{N}}-\bar{N}$ is sufficiently small that q(N) can be approximated by a second order Taylor expansion over the domain $[\bar{\bar{N}},\bar{N}]$.

2. Demand and Surplus

In this section the demand for the natural resource by each of the identical users is considered. The number of users, L, is sufficiently large that each user contributes a negligible impact on quality. Let n_i be the usage of the ith user and $\phi(q(N),n_i)$ be that user's willingness to pay for the n_ith use. The user's consumer surplus is:

$$s_i(n_i) = \int_0^{n_i} \phi(q(N),\nu)d\nu \; - pn_i \qquad (1)$$

where p is the price per use. Write

$$\phi(q(N),n_i) = q(N)w(n_i) \qquad (2)$$

As $q(0)=1$, $w(n_i)$ can be interpreted as the willingness to pay of individual i given the resource has pristine quality. Assume $w(n_i)>0$ for $n_i<\tilde{n}$ and $w(n_i)=0$ for $n_i=\tilde{n}$ and that $w(n_i)$ and $w'(n_i)$ are continuous and differentiable for all $n_i\in[0,\tilde{n}]$. Furthermore $w'(n_i) < 0$ for all $n_i\in[0,\tilde{n}]$. The user's consumer surplus for $n_i\in[0,\tilde{n}]$ is:

$$s_i(n_i) = q(N)b(n_i) - pn_i \qquad (3)$$

where the user's benefit, $b(n_i)$, is:

$$b(n_i) = \int_0^{n_i} w(\nu)d\nu \qquad (4)$$

Observe $b(0) =0, b'(n_i) = w(n_i)>0$ and $b''(n_i)=w'(n_i)<0$ for all $n_i\in[0,\tilde{n}]$. The surplus (3) provides a good approximation of users' welfare when income effects can be treated as negligible, in particular when w_in_i represents a small fraction of users' income (see Tirole, 1988, p. 11).

User i chooses the use level that maximise their consumer surplus. Thus:

$$q(N)w(n_i) = p \qquad (5)$$

Note:

$$s_i''(N) = q(N)w'(n_i) <0 \qquad (6)$$

Thus, faced with a common p and q(N), users choose a common level of use n, i.e. $n_i=n$.

Let:

$$N = \sum_{i=1}^{L} n_i = Ln \qquad (7)$$

where $N \in [0, \tilde{N}]$ and $\tilde{N} \equiv L\tilde{n}$. Thus:

$$q(N)W(N) = p \qquad (8)$$

where $W(N)=w(N/L)$. There is a unique relationship between price and usage level if, for every price, there is one value of $N \in [0, \tilde{N}]$ which satisfies (8). This is the case provided $\frac{d(q(N)W(N))}{dN} < 0$ or:

$$-E_q > E_W \qquad (9)$$

for all $N \in [0, \tilde{N}]$ where $E_W = \frac{NW'(N)}{W(N)}$. Note that this relationship always holds if $q'(N) \leq 0$ for all N. In this event it is possible for pricing to achieve any usage level in $[0, \tilde{N}]$ by using pricing.

3. Social surplus and optimal use

The social surplus is the sum of individual's surplus. Thus, for $N \in [0, \tilde{N}]$:

$$S(N) = \sum_{i=1}^{L} s_i(n) + pn_i = q(N)[Lb(n)] = q(N)B(N) \qquad (10)$$

where $B(N) = \sum_{i=1}^{L} b(n_i) = Lb(n) = Lb(N/L)$ is the social benefit from use. Note $B'(N) = W(N)$ and $B''(N) = W'(N)$. Define $E_B = \frac{NB'(N)}{B(N)}$. It is not possible, *a priori*, to sign $E_B'(N)$

$\equiv \frac{d}{dN}\left(\frac{NB'(N)}{B(N)}\right)$ or $E_B''(N) \equiv \frac{d^2}{dN^2}\left(\frac{NB'(N)}{B(N)}\right)$. Assume $E_B'(N)$ and $E_B''(N)$ are both continuous and differentiable for all $N \in [0,\tilde{N}]$.

Definition 1. The use level in $[0,\tilde{N}]$ which maximises the surplus is termed the globally efficient use level (GEUL). If the surplus attains a local maximum at a given use level in $[0,\tilde{N}]$, that use level is termed a locally efficient use level (LEUL). A locally least efficient use level (LLEUL) is one for which the surplus attains a local minimum.

Hence:

Proposition 1. (a) A LEUL, N*, satisfies either:

$$E_B(N^*) = -E_q(N^*) \text{ and } E_B'(N^*) < -E_q'(N^*) \qquad (11)$$

or $N^*=0$ or $N^*= \tilde{N}$.

(b) LEULs need not be unique.

Proof of Proposition 1 (a). A LEUL, $N \in [0,\tilde{N}]$, could occur in the interior or on the boundary of the domain of S (0 or $\tilde{N}$). The condition for an interior LEUL is found by differentiating the surplus:

$$S'(N) = q'(N)B(N) + q(N)W(N) = \left(\frac{q(N)B(N)}{N}\right)(E_q + E_B) \qquad (12)$$

and:

$$\begin{aligned} S''(N) &= q''(N)B(N) + 2q'(N)W(N) + q(N)W'(N) \\ &= (E_q+E_B)\frac{d}{dN}\left(\frac{q(N)B(N)}{N}\right) + \frac{q(N)B(N)}{N}\frac{d}{dN}(E_q+E_B) \qquad (13) \end{aligned}$$

At a turning point $S'(N) = 0$ and thus $E_B = -E_q$. Therefore at a turning point:

$$S''(N) = \frac{q(N)B(N)}{N} \frac{d}{dN} (E_q + E_B) \tag{14}$$

A turning point is a LEUL if $-E_q' > E_B'$ and a LLEUL if $-E_q' < E_B'$. ||

A LEUL is one where the additional benefit of use is offset by the additional deterioration in quality, i.e. where the elasticity of quality equals the elasticity of the user benefit. The efficient price is set to achieve the efficient level of use according to (8). Figure 2 indicates that (11) need not yield a unique LEUL. Indeed, it is readily concluded from figure 1 that there may be any number of LEULs.

The number of LEULs are shown below to depend on the level of demand. The following definition is required to demonstrate this:

Definition 2: There is said to be (i) very low demand if $0 < \tilde{N} < \underline{N}$, (ii) low demand if $\underline{N} < \tilde{N} < \hat{N}$, (iii) medium demand if $\hat{N} < \tilde{N} < \bar{N}$ and (iv) high demand if $\tilde{N} > \bar{N}$.

The optimal use levels under each level of demand are considered below.

Proposition 2. Assume there is very low demand for the natural resource. Then $\tilde{N}$ is the unique LEUL.

Proof of proposition 2: For all use levels $N \in [0, \tilde{N}]$ the surplus is increasing with N as:

$$S'(N) = q'(N)B(N) + q(N)W(N) > 0 \tag{15}$$

and $q'(N) \geq 0$ and $W(N) > 0$. ||

Changes in the surplus with use occur because quality changes and the user benefit from use changes. User benefit increases with use up to the point where $N = \tilde{N}$, then remains constant. For very low use levels, i.e. $N \in [0, \underline{N}]$, quality is assumed to either increase with use or remain constant with use. Thus, under very low demand, the surplus must be increasing up to the use level $\tilde{N}$.

Proposition 3. If there is positive demand for the level of use $\underline{N}$, i.e. $W(\underline{N})>0$, the GEUL is strictly greater than $\underline{N}$

Proof of proposition 3: If $W(\underline{N})>0$ then $W(N)>0$ for all $N\in[0,\underline{N}]$. Hence $S'(\underline{N}) = q'(N)B(N) + q(N)W(N)>0$ for all $N\in[0,\underline{N})$ and $S'(\underline{N}) = q(\underline{N})W(\underline{N}) > 0$. ||

If demand is positive at $N=\underline{N}$, then user benefit increases with use. When $N =\underline{N}$ quality does not vary with small increases in use. Thus the surplus increases with use when $N =\underline{N}$. Proposition 2 and 3 imply that the efficient use entails some deterioration in quality from the pristine level.

Proposition 4. Assume there is low demand for the natural resource. Then the unique LEUL lies in $(\underline{N},\tilde{N})$.

The proof of proposition 4, and the proof of the propositions below, are given in the appendix. Proposition 4 holds because, under low demand, the surplus is a concave function of N, and thus it has a unique maximum.

Propositions 2 and 4 show that the possibility of multiple LEULs only arises when there is either medium or high demand. The cases of medium and high demand are now analysed. The following definition is used to obtain a limit on the number of LEULs.

Definition 3: Let $F(N) = E_q(N) + E_B(N)$ and $\tilde{D}=(\underline{N}, \tilde{\tilde{N}})$, where $\tilde{\tilde{N}} =\min(\bar{N},\tilde{N})$. Let $\Gamma=\{\Gamma^i\}$be the set of connected subset of $\tilde{D}$ for which $F''(N) \geq 0$ for each element of Γ^i and $F''(N)>0$ for at least one member of Γ^i. Let T^+ be the number of elements of Γ. Let $\Omega=\{\Omega^i\}$ be the set of connected subset of $\tilde{D}$ for which $F''(N) \leq 0$ for each element of Ω^i and $F''(N)<0$ for at least one member of Ω^i. Let T^- be the number of elements of Ω.

T^+ represents the number of intervals in the region of deterioration for which the second derivative of the elasticity of user benefit is greater than the second derivative of the elasticity of quality. Assume (realistically) that T^+ and T^- are finite, even though this need not be the case given the mathematical assumptions above. Intuitively T^+ measures the number of times quality accelerates faster than benefit as use increases over the region of deterioration. Thus T^+ can be interpreted as a measure of the relative volatility of user benefit and quality. The relative volatility places an upper limit on the number of LEULs.

<u>Proposition 5</u>: The number of LEULs in $\tilde{D}$ is less than or equal to T^++1. Furthermore:

(a) If $T^+=T^-+1$, the number of LEULs in $\tilde{D}$ is less than or equal to T^+ and the number of LLEULs is less than or equal to T^+.

(b) If $T^+=T^-$, the number of LEULs in $\tilde{D}$ is less than or equal to T^++1 and the number of LLEULs is less than or equal to T^+.

(c) If $T^+=T^-$-1, the number of LEULs in $\tilde{D}$ is less than or equal to T^++1 and the number of LLEULs is less than or equal to T^+.

The proof of proposition 5 is given in the appendix. Intuitively, proposition 5 holds because two local maximum of the surplus must be separated by a local minimum. At a LLEUL $F'(N)>0$. However, at a LEUL $F'(N)<0$. Thus two LEULs must be separated by a set in which $F''(N)>0$ and another set in which $F''(N)<0$. If there are T^+ sets in which $F''(N)>0$, there are potentially $T^+ + 1$ LEULs. The intuition for parts (a)-(c) of proposition 5 is similar.

In many fragile environments, the fall in quality from its pristine to its minimum level may be very swift once the threshold level of use is past. In this case quality is likely to be more variable than benefit over the region of deterioration. In this case it is appropriate to make the following assumption:

<u>Condition 1</u>. $E''_B(N) > -E''_q(N)$ for all $N \in \tilde{D}$

Alternatively it might be the case that, once the threshold level of use is past the rate of deterioration in quality initially accelerates. Therefore the elasticity of quality is initially more variable than the elasticity of user benefit. However as use increases the rate of deterioration of quality steadies, and the elasticity of user benefit becomes more variable. In this case the following assumption could be made.

<u>Condition 2</u>. $E''_B(N) > -E''_q(N)$ for all $N \in [\underline{N}, \underline{\underline{N}}]$ and $E''_B(N) < -E''_q(N)$ for all $N \in [\underline{\underline{N}}, \tilde{\bar{N}}]$

Alternatively, the deterioration of the quality of the natural resource may be relatively steady or the decline in user benefit may accelerate with use. This may occur in robust environments or where infrastructure is present. In this case the following assumption is appropriate:

<u>Condition 3</u>. $E''_B(N) < -E''_q(N)$ for all $N \in \tilde{D}$

The following two propositions show that the number of local maximum of S(N) is restricted under conditions 1, 2 and 3.

Proposition 6. Assume there is medium demand for the use of the natural resource. Then there is at least one LEUL in $[\underline{N}, \tilde{N}]$. Further,

(a) there is a unique LEUL if condition 1 holds.

(b) there are two LEULs if condition 2 holds

(c) a unique LEUL if condition 3 holds.

Figure 3 shows three examples in which there is medium demand and condition 1, condition 2 and condition 3 holds. Under medium demand, the elasticity of user benefit curve cuts the N axis between $\hat{N}$ and $\bar{N}$ as shown by all the elasticity of user benefit curves in figure 3. At use level $\underline{N}$ the elasticity of user benefit lies above the elasticity of quality (which is equal to zero) and at use level $\tilde{N}$ the elasticity of user benefit (which is zero) lies below the elasticity of quality. Clearly the curves must cross at least once.

If the elasticity of user benefit curve is given by E_B^1 then condition 1 holds. The slope of the elasticity of quality becomes relatively steeper than the elasticity of user benefit as use increases. Thus, under medium demand, there can be only one intersection of the two curves. Similarly, if the elasticity of user benefit curve is given by E_B^3 then condition 3 holds. The slope of the elasticity of user benefit becomes relatively steeper than the elasticity of user benefit as use increases. Again there can be only one intersection of the two curves. Finally, the curve E_B^2 shows how there may be two LEULs in D under condition 2.

Proposition 7. Assume there is high demand for the use of the natural resource. Then $\tilde{N}$ is a LEUL. (a) There may be another LEUL in D under either condition 1 or condition 2. (b) $\tilde{N}$ is the unique LEUL under condition 3.

Figure 4 shows three examples in which there is high demand and condition 1, condition 2 and condition 3 holds. Under high demand, the elasticity of user benefit curve cuts the N above $\bar{N}$ as shown by all the elasticity of user benefit curves in figure 4. As N approaches $\tilde{N}$ from below the elasticity of user benefit is positive while the elasticity of quality is zero. Hence $\tilde{N}$ is a LEUL.

The elasticity of user benefit may cut the elasticity of quality curve in D. If the elasticity of user benefit curve is given by E_B^1 then condition 1 holds. The slope of the elasticity of quality curve becomes steeper relative to the elasticity of user benefit curve as use increases. Thus, as depicted in figure 4, there can be maximum of two intersection of the two curves. The intersection with the lower use level is the LEUL.

Under condition 3 the slope of the elasticity of user benefit, E_B^3 , becomes steeper relative to the elasticity of user benefit as use increases. Hence, under high demand, there cannot be any intersection of the two curves in D. Finally, the curve E_B^2 shows how there can be at most one LEUL in D under high demand and condition 2.

If the two curves, E_B^1 and E_B^2 , in figure 4 were shifted vertically upward they would still satisfy condition 1 and condition 2 respectively. If this vertical shift was sufficiently great, they would be no cut the elasticity of quality curve. In this case $\tilde{N}$ would be the unique local maximum.

Suppose $\tilde{N}$ and $N_D^* \in D$ are both LEULs. Then $\tilde{N}$ is the GEUL if:

$$\frac{B(\tilde{N})}{B(N_D^*)} > \frac{q(N_D^*)}{q(\tilde{N})} \qquad (16)$$

That is, use level $\tilde{N}$ is more efficient than use level N_D^* if the increase in user benefit outweighs the loss in quality. In this event it is not necessary to impose a fee to achieve the globally efficient level of use. If the inequality sign in (16) is reversed the use level N_D^* is more efficient than use level $\tilde{N}$. From (8), this level of use can be achieved by setting price $p^* = q(N_D^*)W(N_D^*)$.

Some recreational uses of a natural resource cause environmental damage which imposes a cost on non-users.[1] Suppose the recreational use of the natural resource imposes a cost on non users, C, which is independent of the level of use. Let N_G^* be the globally efficient level of use. Then recreational use of the natural resource is efficient only if:

$$C < q(N_G^*)B(N_G^*) \qquad (17)$$

If the inequality in (17) is reversed the cost created by recreational use to non-users outweighs the recreation benefit to users.

4. Changes in demand

In practice, changes in demand for the natural resource could arise because of changes in real income, changes in the population or loss of substitute activities. An increase in real income increases existing users' willingness to pay while an increase in the population increases the number of users. A loss of substitute activities increases the demand for use at each level of willingness to pay.

An increase in real income could impact on a user's willingness to pay in many ways. However assume that the growth in real income brings about a proportional change in each users' willingness to pay for each use. User benefit after change in income can be written as:

$$b(n_i) = \alpha b_o(n_i) \qquad (18)$$

where $b_o(n_i)$ is the initial user benefit function and α represents the change in the willingness to pay. The parameter α is proportional to the growth in income if users have homothetic preferences.

Proposition 8. The GEUL is independent of the growth in benefit per use, and the efficient price is proportional to the growth in benefit per use.

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[1] For example, the recreational use of a pristine area may cause it to lose its status as a wilderness area.

An increase in the number of users changes the elasticity of user benefit curve and thus the efficient level of use. Specifically, the impact of a change in the number of users on the elasticity of user benefit if found by writing:

$$E_B(N) = \frac{Nb'(N/L)}{Lb(N/L)}. \qquad (19)$$

Before using (19) to assess the impact of a change in the number of users, consider the impact of an increase the number of uses demanded at each level of willingness to pay. This impact can be captured by writing:

$$E_B(N) = \frac{NB'(N/\beta)}{\beta B(N/\beta)}. \qquad (20)$$

where β is proportionate increase in use at each level of willingness to pay. By comparing (19) and (20) it can be seen that the formal analyses of an increase in the number of users and an increase in the demand at a given level of willingness to pay are equivalent. For brevity, the discussed below is conduction only in terms of a change in the number of users.

Proposition 9. Let $N_D^* \in D$ be a LEUL. An increase in the number of users:

(a) increases (decreases) the LEUL and decreases (increases) quality if $E_B'(N_D^*)<(>)0$,

(b) decreases (increases) the locally efficient use per person and increases (decreases) the price if $E_q'(N_D^*) > (<) 0$,

(c) increases the surplus, but by less than the surplus per user.

The proof of proposition 9 is given in the appendix. From (19) a unit increase in L shifts the elasticity of user benefit one unit to the right. If the elasticity of user benefit curve is downward sloping and the elasticity of quality curve is upward sloping, as shown by the curve E_B^M in figure 5, an increase in the number of users results in an increase in the LEUL. Because the elasticity of quality curve is upward sloping at the LEUL, the increase in total use is less than the increase in users. Therefore the use per person decreases. To achieve this decrease, the locally efficient price must increase. An increase in the number of users increases user benefit and hence the surplus. However, the surplus per user declines because each user is making less use of the natural resource and quality is declining with increased total use.

Under high demand $\tilde{N}$ is a LEUL. An increase in the number of users increases the use level by the same proportion. In this case the surplus may be written as:

$$S(\tilde{N}) = Ls(\tilde{n}) = q(\tilde{N})Lb(\tilde{n}) \tag{21}$$

Hence the surplus also increases by the same proportion as the increase in users. Thus:

$$\frac{\partial S}{\partial L} = q(\tilde{N})b(\tilde{n}) = S(\tilde{N})/L > 0 \tag{22}$$

An increase in the number of users increases the surplus by the surplus per user. As the number of users increases, quality does not vary, and each user consumers a constant amount $(\tilde{n})$ of the natural resource.

Define the minimum number of users that can generate high demand as L_o, i.e. $L_o\tilde{n}$ $= \bar{N}$.

Proposition 10. Suppose that, under high demand, there is one LEUL $N_D^* \in D$. If L is in the neighbourhood of L_o, then N_D^* is the global maximum. Furthermore there exists a number of users, L^c, where if $L<L^c$ then N_D^* is the GEUL and if $L>L^c$ then $\tilde{N}$ is the GEUL.

Consider the elasticity of user benefit and elasticity of quality curves depicted in figure 5. These curves satisfy condition 1, and thus proposition 7 shows that there are potentially two LEULs under high demand. The curve E_B^{MH} in figure 5 represents an elasticity of benefit curve when $L=L_o$. In this case $N = \tilde{N}$ $(=\bar{N})$ is a local minimum of the surplus and thus N_{MH}^* is the GEUL. An increase in the number of users shifts the elasticity of user benefit curve to the right and thus $N = \tilde{N}$ becomes a LEUL. However if L is arbitrarily close to L_o then the surplus is arbitrarily close to $S(\bar{N})$. Hence the use level N_D^* is more efficient than $\tilde{N}$.

Now consider the elasticity of user benefit curve E_B^{HT} in figure 5. This curve represents the number of users, L_T, which is high enough that the elasticity of user benefit and elasticity of quality curves are tangent to each other. At the use level where the curves touch, N_{HT}^*, the surplus has a point of inflection. Thus $\tilde{N}_{HT}$ is the GEUL. There exists a number of users L^c, where $L_o<L^c<L_T$, corresponding to the elasticity of user benefit curve E_B^{Hc} which generates LEULs N_{Hc}^* and $\tilde{N}_{Hc}$ which are equally efficient. If the number of

users is less than L^c, the GEUL lies within the region of deterioration. If the number of users is greater than L^c the GEUL is greater than $\bar{N}$.

In summary, an increase in the number of users has the following effect on the GEUL. If initially there is low or medium demand, an increase in the number of users causes the use per person to fall (rise) and the price to rise (fall) if $E_q'(N_D^*)>(<)0$. In addition total use rises (falls) and quality falls (rises) if $E_B'(N_D^*)<(>)0$. Increasing the number of users sufficiently creates high demand. At some stage the GEUL switches from being in the region of deterioration to $\tilde{N}$. At this point the efficient price drops to zero, the use per person increases to $\tilde{n}$, and the quality drops to its minimum level. Further increases in demand increases the GEUL, but does not change the price, use per person or quality.

Finally consider the case in which the recreational use of the natural resource imposes a cost, c, on each non user. Suppose the number of non-users is growing at the same rate as users. Specifically, let the number of non users be λL. Then the total cost of the recreational use of the natural resource on non-users is L. The cost of the recreational use of the natural resource grows in proportion to the number of users. However, it is shown above that the surplus per user is decreasing. Thus it is possible that as the number of users (or population) increases it is efficient use can switch from a positive level to zero. This could only be the case if $c\lambda>q(\tilde{N})b(\tilde{n})$, that is the cost to non-users outweighs the benefits of use when the natural resource fully deteriorated. If $c\lambda<q(\tilde{N})b(\tilde{n})$ it is efficient to use the natural resource for recreation.

5. Conclusion

This paper considered the efficient recreational use of a natural resource. It is shown that the social surplus arising from the use of the natural resource may not have a unique local maximum. Indeed, the surplus could have an arbitrary number of local maxima. A condition is presented that provides an upper bound on the number of local maxima given the quality and benefit functions. This number is related to the relative volatility of the two functions. Using this condition it is shown that, in a variety of circumstances that are likely to hold in practice, the number of local maxima is less than or equal to two. Where the surplus has two local maxima, the higher use level is the efficient level if the fall in quality that occurs when moving from the low to high use level is offset by the increased benefit in use.

These results are used to analyse the impact on the efficient use level of an increase in demand for the recreational use of the natural resource. It is argued that an increase in demand arising from increased demand per person (such as might occur if there is an increase in users' income) does not change the efficient level of use. However an increase in the number of users, or a loss of substitute activities, changes the efficient use level. If demand is either at a low or medium level, an incremental increase in the number of users will change the efficient fee and reduce the surplus per user. A sufficiently large increase in the number of users creates high demand. A 'high' level of use, in which the fee is zero and quality is at its minimum level, is locally efficient. If the number of users is sufficiently great the high level of use is the globally efficient level.

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Mathematical Appendix to:

THE EFFICIENT RECREATIONAL USE OF A NATURAL RESOURCE

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Appendix 1: Quadratic approximation of $E_q(N)$

In this appendix it is shown that $E_q(N)$ has a unique minimum if it can be approximated by a second order Taylor expansion. Taylor's theorem is used to obtain the following second order approximations:

$$q(N) \approx q(\bar{N}) + q'(\bar{N})\delta N + q''(\bar{N})(\delta N)^2 = q(\bar{N}) + ½\, q''(\bar{N})(\delta N)^2$$

and:

$$q'(N) \approx q'(\bar{N}) + q''(\bar{N})\delta N + ½\, q'''(\bar{N})(\delta N)^2 = q''(\bar{N})\delta N + ½\, q'''(\bar{N})(\delta N)^2$$

where $\delta N = N\text{-}\bar{N}$. Observe that, as $q''(\bar{N})>0$ then, for $q'(N)$ to have a unique minimum, it must be the case that $q'''(\bar{N})>0$. Then:

$$E_q(N) \approx \frac{(\bar{N}+\delta N)\,(q''(\bar{N})\delta N + ½\, q'''(\bar{N})(\delta N)^2)}{q(\bar{N}) + ½\, q''(\bar{N})(\delta N)^2}$$

$$\approx \left(\frac{\bar{N}\delta N}{q(\bar{N})}\right)\left(\frac{q''(\bar{N})}{2} + \left(q''' + \frac{q''(\bar{N})}{\bar{N}}\right)\delta N\right)$$

$E_q(N)$ has a unique minimum when:

$$\delta N = \frac{-\,\bar{N}q''(\bar{N})}{2(q''(\bar{N}) + \bar{N}q'''(\bar{N}))} < 0$$

or $$\bar{\bar{N}} = \bar{N} - \frac{\bar{N}q''(\bar{N})}{2(q''(\bar{N}) + \bar{N}q'''(\bar{N}))}$$

Appendix 2: Proof of proposition 4

Over the range $N\in[\underline{N},\tilde{N})$ $S'(N)$ has an indeterminate sign, but $S'(\underline{N})=W(\underline{N})>0$ and $S'(\tilde{N})$ $=q'(\tilde{N})B(\tilde{N})<0$ and:

$$S''(N) = q''(N)B(N) + 2q'(N)W(N) + q(N)W'(N)<0 \qquad (23)$$

Thus $S(N)$ must reach a maximum for a use level in $(\underline{N},\tilde{N})$. ||

Appendix 3: Lemma 1

The following lemma is used in the proof of proposition 5.

> <u>Lemma 1:</u> (a) Under medium demand the number of LEULs in $\tilde{D}$ equals the number of LLEULs in $\tilde{D}$ plus one. (b) Under high demand the number of LEULs in $\tilde{D}$ equal the number of LLEULs in $\tilde{D}$.

Proof of lemma 1: (a) As $E_B(\underline{N}) > -E_q(\underline{N})=0$ and $E_B(\tilde{N})=0 <-E_q(\tilde{N})$, the E_B and $-E_q$ curves must cross an odd number of times. Denote the number of intersections of E_B and $-E_q$ as 2m-1, where m is a positive integer. Denote the use levels at which these intersections occur as N^1, N_1 ,N^2 ,N_2 ,....N_{m-1},N^m, where $N^1< N_1<N^2<N_2<....N_{m-1}<N^m$. N^1 and N^m must be LEULs, and thus are N^1, N^2,....,N^m. Thus N_1, N_2,...., N_{m-1} are LLEULs. (Under medium demand $\tilde{N}$ is a LLEUL. However $\tilde{N}\notin \tilde{D}$.)

(b) As $E_B(\underline{N}) > -E_q(\underline{N})=0$ and $E_B(\bar{N}) >-E_q(\bar{N})=0$, the E_B and $-E_q$ curves must cross an even number of times. N^1, N_1 ,N^2 ,N_2 ,.... ,N^{m-1}, N_{m-1}. N^1 must be a LEUL and N_{m-1} must be a LLEUL. ||

Appendix 4: Proof of proposition 5

The proof is by contradiction. Suppose that use levels, $N^i \in \tilde{D}$ i=1,2,...m, where $m>T^+ +1$, are LEULs, i.e. $F(N^i)=0$ and $F'(N^i)<0$. Suppose, without loss of generality, that $N^i<N^j$ for $i<j$. Observe that there must exist a use level, N_i, where $F(N_i)=0$ and $F'(N_i)>0$ and $N^i< N_i <N^{i+1}$ for i =1, 2,..,m-1. (Note that if $\tilde{N}<\bar{N}$, then $\tilde{N}$ is a LLEUL. However $\tilde{N}\not\in \tilde{D}$.) There must therefore exist at least one connected set of use levels, $\Gamma^i\subset [N_{i-1},N^{i+1}]$, i=1,2,..m for which $F''(N)\geq 0$ for each use level in Γ^i and $F''(N)>0$ for at least one use level in Γ^i. Observe that sets Γ^i and Γ^j $i\neq j$ are disjoint, as there must also exist the sets $\Omega^i\subset[N^{i-1},N_i]$, i=1,2,..m-1 for which $F''(N)\leq 0$ for each use level in Ω^i and $F''(N)<0$ for at least one use level in Ω^i. It is assumed that the number of sets Γ^i is T^+, thus $T^+>m-1$. This is a contradiction, hence the number of local maximum is less than or equal to $T^+ +1$.

(a) This result is shown by contradiction. Assume $m^+>T^+$ is the number of LEULs and m^- is the number of LLEULs. As shown above, each LEUL must be separated by at least one set, Ω^i, where i=1,2,...m^+-1. By assumption the number of such sets is T^+-1. Hence $m^+-1\leq T^+-1$, or $m^+\leq T^+$. This is a contradiction, hence m^+ is not greater than T^+. Observe that, by lemma 1, $m^+=m^- +1$ or $m^+=m^-$. Thus the number of local minimum is less than T^+.

The proof of parts (b) and (c) are similar to above. ||

Appendix 5: Proof of proposition 6

As $E_B(\underline{N}) > -E_q(\underline{N})=0$ and $E_B(\tilde{N})=0 < -E_q(\tilde{N})$, there must be one LEUL, satisfying (11), in $[\underline{N}, \tilde{N}]$. (a) Under condition 1 $T^+=1$ and $T^-=0$. By proposition 5(a) the number of LEULs is less than or equal to 1. (b) Under condition 2 $T^+=1$ and $T^-=1$. Thus, by proposition 5(b), the number of LEULs is less than or equal to 2. (c) Under condition 3 $T^+=0$ and $T^-=1$. By proposition 5(c) the number of LEULs is less than or equal to 1. ||

Appendix 6: Proof of Proposition 7

(a) Condition 1 as for proposition 6(a). Under condition 2, Proposition 5(d) shows the number of LEULs is less than 2 and the number of LLEULs is less than 1. Lemma 1 thus implies that the number of LEULs is less than 1. (b) Under condition 3, proposition 5(c) shows the number of LEULs is less than 1 and the number of LLEULs is zero. Therefore lemma 1 implies that number of LEULs in D is zero. ||

Appendix 7: Proof of proposition 8

Observe that the elasticity of user benefit is independent of α. Thus the use level at which local maximum of the surplus occur are unaffected by changes in α. Furthermore, the ratio of user benefit at different local maximum of the surplus, on the right hand side of (16), is also unaffected by changes in α. Thus the GEUL is unaffected by changes in α. However the efficient price is given by:

$$p = \alpha q(N^*)W_o(N^*/L) \qquad (24)$$

Thus an increase in users' willingness to pay increases the efficient price. ||

Appendix 8: Proof of proposition 9

It is shown above that under low, medium and high demand the GEUL may lie in the domain D. Assume this is the case. The elasticity of user benefit at the GEUL is:

$$E_B(N_D^*) = \frac{N_D^* b'(N_D^*/L)}{Lb(N_D^*/L)}. \qquad (25)$$

Observe that $\partial E_B/\partial L$ has the opposite sign to $\partial E_B/\partial N$. From the first order condition in proposition 1(a):

$$\partial E_B/\partial L\, dL + (\partial E_B/\partial N)dN_D^* = -(\partial E_q/\partial N)\, dN_D^* \qquad (26)$$

or

$$\frac{d\,N_D^*}{dL} = \frac{-\,\partial E_B/\partial L}{\partial E_B/\partial N + \partial E_q/\partial N} \qquad (27)$$

The second order conditions require that the denominator is negative. Thus the right hand side of (27) is less than one. Thus, the increase in use is less than the increase in the number of users. Therefore the use per person decreases. For this to occur price must increase. Observe that an increase in the number of users increases the GEUL if the elasticity of user benefit curve is downward sloping at the point of intersection with the elasticity of quality curve.

The impact of an increase in the number of users on the social surplus is found by writing:

$$S(N_D^*) = q(N_D^*)Lb(N_D^*/L) \qquad (28)$$

Thus, by the envelope theorem:

$$\frac{dS(N_D^*)}{dL} = \frac{\partial S(N_D^*)}{\partial N}\frac{dN}{dL} + \frac{\partial S(N_D^*)}{\partial L} = \frac{\partial S(N_D^*)}{\partial L} \qquad (29)$$

where:

$$\frac{\partial S}{\partial L} = (S(N_D^*)/L)(1\text{-}E_B(N_D^*))>0 \qquad (30)$$

Hence an increase in the number of uses increase the surplus, but by less than the surplus per user. Thus the surplus per user must be declining. This occurs because each user is making less use of the natural resource and, if $E_B'(N_D^*)<0$, quality is declining with increased total use.

Appendix 9: Proof of proposition 10

Observe that, if L= L_o then $\tilde{N} = \bar{N}$ and $\frac{S(\tilde{N})}{S(N_D^*)} < 1$. Thus N_D^* is the GEUL. From the analysis above it is readily shown that:

$$\frac{d}{dL}\left(\frac{S(\tilde{N})}{S(N_D^*)}\right) = \frac{S(\tilde{N})E_B(N_D^*)}{LS(N_D^*)} > 0 \qquad (31)$$

It is left to show that $\frac{S(\tilde{N})}{S(N_D^*)} > 1$ if L becomes sufficiently large. Observe that :

$$\frac{S(\tilde{N})}{L} = q(\bar{N})b(\tilde{n}) \qquad (32)$$

for all L. However:

$$\frac{S(N_D^*)}{L} = q(N_D^*)b(N_D^*/L) \leq q(\underline{N})b(\bar{N}/L) \qquad (33)$$

Note $b(\bar{N}/L) \rightarrow 0$ as $L \rightarrow \infty$. Thus $\frac{S(N_D^*)}{L} \rightarrow 0$ as $L \rightarrow \infty$. Hence $\frac{S(\tilde{N})}{S(N_D^*)} \rightarrow \infty$ as $L \rightarrow \infty$.

Hence the GEUL switches from the domain D to $\tilde{N}$ as L increases (from an initial level of L_o). ||