Example – Simultaneous Equations

If you buy 4 coffees and 3 teas and the total cost is $18, that is not enough information to work out the cost of one coffee. There are a number of possibilities ranging from

Coffee $4.50 and Tea is free (unlikely but possible)
To
Coffee is free and Tea $6 (also unlikely but possible)
The correct answer would be somewhere in between.

If you go back the next day and buy 3 coffees and 2 teas and the total cost is $13 then YOU have more information and can work out the correct cost of each.

Looking at these as equations where C is the cost of one coffee and T is the cost of one tea:

Day 1
$4C + 3T = 18$

Day 2
$3C + 2T = 13$

With a bit of trial and error you can see that Coffee $3 and Tea $2 is the only solution that works in both cases.

Try it and see.

Solutions

This is why they are called Simultaneous Equations because both must be true at the same time.

This is a simple example but you can use algebraic methods to solve much more difficult problems of this type.

There are three methods of solution:

- Substitution of one variable for another.
- Elimination of one of the variables
- Graphing the two equations to see where they intersect.

A detailed solution for the Tea and Coffee problem looks like this:

Method 1 - Substitution of one variable for another.
Equation 1 $4C + 3T = 18$

Equation 2 $3C + 2T = 13$

Rearrange Equation 2 to find T in terms of C

$$2T = 13 - 3C$$
$$T = (13 - 3C)/2$$

Substitute for T in Equation 1

$$4C + 3(13 - 3C)/2 = 18$$
$$4C + (39 - 9C)/2 = 18$$

(See why Order of Operations is so important)

$$4C + 19.5 - 4.5C = 18$$
$$4C - 4.5C = 18 - 19.5$$
$$-0.5C = -1.5$$
$$0.5C = 1.5$$

In other words half a cup of coffee is $1.50

$$C = 3$$

Using that value in Equation 1 (you could use Equation 2 if you wish)

$$4C + 3T = 18$$

C = 3 so $12 + 3T = 18$

$$3T = 18 - 12$$
$$3T = 6$$
$$T = 2$$

So Coffee costs $3 and Tea costs $2

Method 2 - Elimination of one of the variables

Equation 1 $4C + 3T = 18$

Equation 2 $3C + 2T = 13$

You need to get the same number of one variable in each equation so that you can subtract them and only be left with one variable.

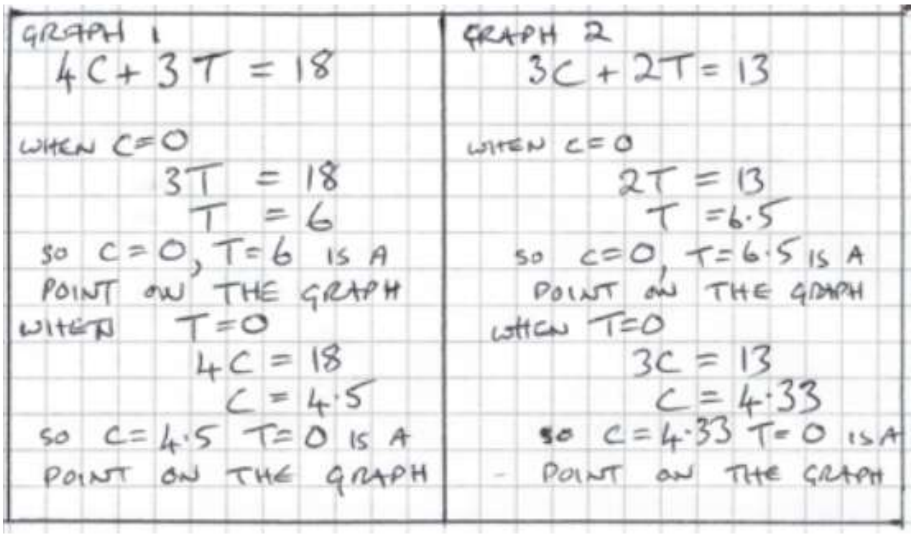
The choices are:
Multiply equation one by 3 and equation two by 4 to be able to get rid of C
or
Multiply equation one by 2 and equation two by 3 to be able to get rid of T
There isn't much in it but the second option is a bit easier

Equation 1 multiplied by 2 becomes $8C + 6T = 36$
Equation 2 multiplied by 3 becomes $9C + 6T = 39$
Subtract equation 1 from equation 2 $C + 0 = 3$
A coffee costs $3
Then substitute into an original equation to find T as in the first method.

Method 3 - Graphing the two equations to see where they intersect.

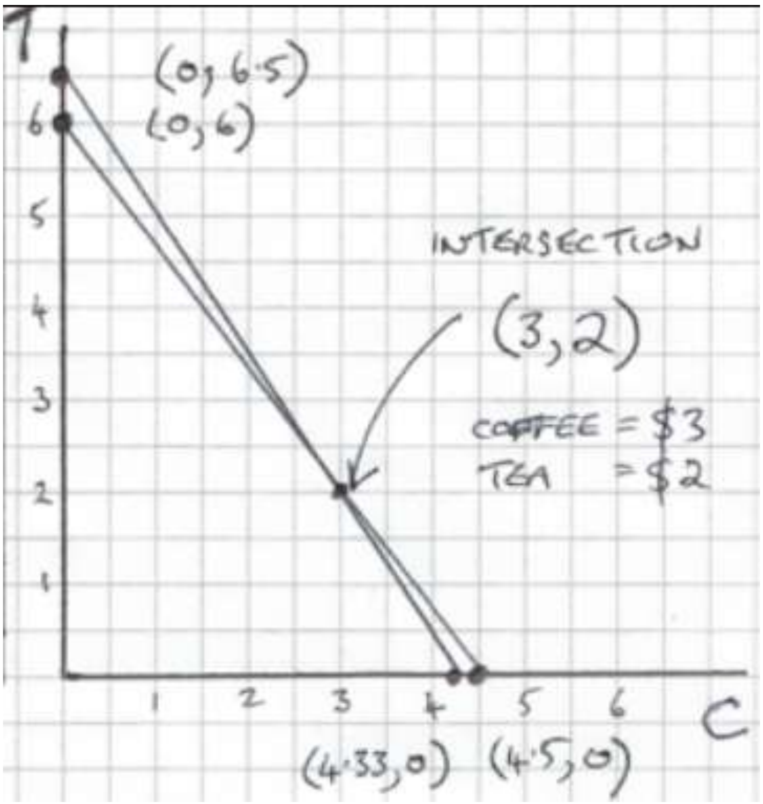
The first step is to find the points where the lines cross the axes by making C and T equal to zero in turns. This is like saying if the coffee was free how much would the tea cost and if the tea was free how much would the coffee cost.

Graph 1 is all the possible solutions for Day1 and Graph 2 is all the possible solutions for Day 2. Only one pair of prices can be true on both days.



GRAPH 1	GRAPH 2
$4C + 3T = 18$	$3C + 2T = 13$
WHEN $C = 0$	WHEN $C = 0$
$3T = 18$	$2T = 13$
$T = 6$	$T = 6.5$
SO $C = 0, T = 6$ IS A POINT ON THE GRAPH	SO $C = 0, T = 6.5$ IS A POINT ON THE GRAPH
WHEN $T = 0$	WHEN $T = 0$
$4C = 18$	$3C = 13$
$C = 4.5$	$C = 4.33$
SO $C = 4.5$ $T = 0$ IS A POINT ON THE GRAPH	SO $C = 4.33$ $T = 0$ IS A POINT ON THE GRAPH

Then use those points to draw the lines. The vertical axis is the cost of tea and the horizontal axis is the cost of coffee.



The point of intersection (3,2) is the only pair of answers where both statements can be true at the same time.