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Menus of Linear Contracts in Procurement with Type-Dependent Reservation Utility

Shane B. Evans

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Shane B. Evans*

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Abstract

This paper examines the influence of type-dependent reservation utility on the optimality of linear contracts in a Principal-Agent model of procurement. Type-dependency of reservation utility, combined with the requirements of individual rationality and incentive compatibility in the principal's contracts induces a countervailing incentive effect, the strength of which depends on an index of quality or degree of competition that the agent would face in an external private market. The results show how the curvature of the reservation utility dictates whether the optimal contracts can be implemented with a menu of linear contracts, and how the magnitude of the private market index influences the net-transfer rule.

Key words: Linear Contracts, Countervailing Incentives, Type Dependent Reservation Utility
JEL Classification: D82, D86, H57

*School of Economics & Finance, Commerce Building, University of Tasmania, Hobart TAS 7001, Australia. Email: shane.evans@utas.edu.au. This paper has benefited from the feedback and advice from Martin Richardson, Ron Stauber, Aki Asano, Phong Ngo, Chris Jones, and Jim Butler. Thanks also to Bernard Salanié and Stephen King for comments on an earlier draft.

1 Introduction

Procurement contracts are written in complex institutional environments, and where the procurer is often at an informational disadvantage to the firm with which it wishes to contract. As such, procurement contracts usually involve a sophisticated rule for transferring payments that are contingent on the observed project cost. Moreover, the transfer rule must embody the opportunistic behaviour of the contracted firm. The complexity of these optimal contracts presents not just a theoretical problem, but a practical one since complex contracts are not typically used by procurement agencies. To overcome some of the complexity, one line of research beginning with Laffont and Tirole (1986) has explored the feasibility of implementing procurement policy with a menu of contracts that are linear in cost over-runs.[1] The generally accepted result is that implementation with a menu of linear contracts is feasible provided the transfer rule is decreasing and convex in the project cost. This is because the rule can then be replaced with a menu of its tangents, each tangent representing a linear contract. Hence, Laffont and Tirole's linearity property is important because it connects the theoretically complex contracts to the observation that procurement contracts are typically linear in cost over-runs.

The regularity conditions for linear contracts to be feasible are well established.[2] However there is one aspect of procurement environments for which the linearity property has not been tested: when the contracted firm has type-dependent reservation utility. In most models of procurement, the reservation utility, or outside opportunity of the contracted firm is normalised to zero. However, relaxing this assumption to allow for type-dependent reservation utility affects the core of the agency problem faced by the procurer: the incentives facing the contracted firm allow for more complicated opportunistic behaviour. This paper attempts to extend the linearity property to include such environments.

Normalisation of reservation utility across different agent types is not a desirable property to impose on procurement models in many cases. For example, in Australia, the Commonwealth government procures pharmaceuticals under a policy known as the Pharmaceutical

[1] A larger body of research simply assumes this linearity property as a starting point for their work. See, for example, the recent work of Rogerson (2003) and Chu and Sappington (2007).

[2] Rogerson (1997) spells out the regularity conditions in detail.

Benefit Scheme. A pharmaceutical company can make an application to the government to be placed on a registered list, whereupon it can operate in a regulated prescription market. However, the pharmaceutical company could also choose to sell its drugs into a private prescription market, and record a profit that may depend on not just its cost of production, but also the quality of its drug, or the degree of competition it faces or other market variables. Another example is defense procurement: firms in the defense industry typically operate in many international markets, and when using their resources in one contract, have given up contracts in other markets. Again, the size of the profit given up by a firm depends on its production costs as well features of the market that it could have operated in. The problem with allowing for type-dependent reservation utilities is that the principal must compensate the agent for its opportunity cost. This introduces scope for opportunistic behaviour by the agent: by misreporting its type, the agent may receive more compensation for its reservation utility than is warranted.

To be more precise, in Laffont and Tirole's formulation the total observed cost of the project is $C = \theta - e$ where θ is the agent's privately known intrinsic cost of the project and e is the effort that it exerts to reduce cost. Hence, the principal cannot determine whether a high total cost is the result of a high cost agent or a low cost agent not exerting effort. The agent prefers to exert less effort, as it is personally costly, with disutility represented by $\psi(e)$. Moreover, effort is unobservable and non-verifiable, and therefore cannot be written into a contract or enforced by a court of law. The principal must therefore design its contracts to deal with adverse selection from the agent's private information about its intrinsic cost θ, and moral hazard from the principal's inability to observe and verify the agent's effort. So the principal's contracts must incentivise the agent to both report the truth about its intrinsic cost, and to exert a certain level of effort.

To do this, the principal's contracts are based on a report from the agent about its privately observed intrinsic cost. After receiving the report θ, the principal reimburses the cost $C(\theta)$ to the agent, and transfers an amount $y(\theta)$. Hence, the agent's payoff is $y(\theta) - \psi(e)$. Ideally, this transfer just covers the agent's opportunity cost and its disutility from effort. However, since effort is unobservable and non-verifiable, a lower cost agent has incentive to over-report its intrinsic cost to receive the transfer designed for a higher cost agent. By doing so, the lower cost agent earns a rent since it can reduce its effort and save on disutility. This is the usual direct effort incentive effect. To reduce the expected rent that the principal has to pay out, it may design its contracts in a way that reduces the effort it requires of higher cost agents. This in turn reduces the reward for lower cost agents to over-report. In doing this, the principal therefore trades of rent extraction against a reduction in the efficient level of effort in cost reduction.

This paper explores the additional incentive effect that arises when the agent's reservation utility depends on its cost type. The transfer $y(\theta)$ must cover not only the disutility of the agent's effort, but must also account for its reservation utility as well. If the reservation utility decreases in the agent's type, then a higher cost agent has incentive to under-report its cost and receive the higher transfer designed for a lower cost agent. This effect works in the opposite direction to the direct effort incentive, hence it is known as the countervailing incentive effect. This time, to reduce the expected rent that the principal has to pay out to induce truthful reporting, the contracts require agents to exert more effort which increases the disutility from under-reporting.

The balance of the opposing incentive effects depends on the magnitude of the agent's reservation utility, which in turn depends on the characteristics of the alternative market in which it could have operated. Private market features, like drug or weapon quality or fierceness of competition in the private market, affect the magnitude of the agent's reservation utility and hence the importance of the countervailing incentive in the principal's problem. Hence, these market features are summarised by a "private market parameter", indexed by the real-valued parameter $s \in \mathbb{R}_+$. This enables different strengths of the countervailing incentive effect on the principal's contracts to be explored.

The results show that feasibility of implementation with linear contracts rests on the curvature of the reservation utility in the agent's cost type. It will be shown that provided the reservation utility satisfies a convexity bound, Laffont and Tirole's linearity property is preserved. However, if the reservation utility is concave, an equilibrium menu of contracts may exist, but cannot be implemented with a menu of linear contracts. The paper also demonstrates how different levels of the private market parameter change the strength of the countervailing incentive effect, but do not disturb the linearity property.

The next section reviews some of the relevant literature in procurement and type dependent reservation utility. Then the feasibility of implementation with linear contracts is studied by constructing the equilibrium contracts and determining the conditions on the curvature of the reservation utility that lead to convex net-transfer rules. The last section concludes.

2 Relation to the Literature

The optimality of menus of linear contracts has in the past been a subject of intensive study. Many models of procurement assume from the outset that contracts are linear, or use a linear menu of contracts as a benchmark for the relative performance of their incentive schemes, including some very recent research (see for example, Baron and Besanko (1988), Bower (1993), Rogerson (2003), Chu and Sappington (2007)). The Laffont and Tirole (1986) procurement model first provided the sufficiency conditions for an optimal menu of contracts to be implemented with a menu of linear contracts. They showed that provided the net-transfer rule for procurement is decreasing and convex, then it can be replaced with a menu of its tangents, where each tangent represents a linear contract.

Laffont and Tirole's assertion was followed by a series of research papers directed at finding a generalisation of the linearity property of optimal contracts, as in McAfee and McMillan (1987), Melumad and Riechelstein (1989) and Rogerson (1987). The results from that line of research showed that the optimality of a menu of linear contracts does not hold in general. As qualified in Laffont and Tirole (1993, p. 107):

> *"We thus should not consider the linearity result as a general rule but rather as defining a class of environments in which one can conveniently work with linear schemes."*

However, so far the environments studied in which the linearity property holds have been limited to the case where the reservation utility of each type of agent is normalised to zero. The task of this paper is devoted to re-examining the linearity property in a procurement environment where the reservation utility of the agent is type dependent. It is well known that type dependent reservation utility introduces a new problem, where an agent may no longer have a systematic incentive to always under-report its cost. Hence, the monotonicity property required to properly implement separating contracts may become violated. Problems of this sort have come to be known as countervailing incentives.

In a paper closely related to this one, Lewis and Sappington (1989) study how countervailing incentives can arise in the seminal incentive regulation model of Baron and Myerson (1982). In their framework, adverse selection arises since the principal faces an agent with private information about its marginal production cost. In the equilibrium contracts, the principal trades off efficiency of production against limiting the information rents to low marginal cost types of agents. It does this by distorting downward the quantity it asks high marginal cost agents to produce, thus reducing the benefit to low marginal cost agents from over-reporting their cost. Lewis and Sappington's (1989) extension analyses the case where countervailing incentives are created through the existence of a type-dependent fixed cost of production.

In their model, lower marginal costs are associated with higher fixed costs - so that reservation utility is decreasing in the agent's type. As such, agents no longer have a systematic incentive to over-report their type as in Baron and Myerson. By under-reporting, the agent can be rewarded with a higher transfer to cover its perceived higher fixed cost. However, to ensure their problem satisfies the second order conditions for an optimum, Lewis and Sappington assume that the reservation utility is concave. This assumption leads to the result that their equilibrium contracts entail some pooling: for intermediate cost realisations, the principal induces the same level of production from those cost type agents.

Maggi and Rodriguez-Clare (1995) generalise the countervailing incentives problem in agency contracts. In doing so, they are able to explain the pooling equilibrium result of Lewis and Sappington. They demonstrate that whether an equilibrium contract exhibits pooling or is separating depends on the assumptions placed on the curvature of the reservation utility. Specifically, whenever the reservation utility is concave, pooling characterises the equilibrium contract - as in Lewis and Sappington. However, when the reservation utility is strictly convex, the equilibrium contracts are fully separating.

Maggi and Rodriguez's optimal control technique is employed in this paper in the context of Laffont and Tirole's (1986) procurement model, which features both adverse selection and moral hazard. Their solution employs a rent extraction-efficiency trade off that is similar to the Baron-Myerson result, but involves an inefficiency in cost-reducing effort rather than production.[3] This paper introduces countervailing incentives to the Laffont-Tirole model in a similar manner to Lewis and Sappington: lower intrinsic cost agents have higher reservation utility since they would be more profitable in the private market. However, unlike Lewis and Sappington, no restriction is made from the outset on the curvature of the reservation utility. Rather, this paper derives the restrictions on the curvature that are required for the linearity property to hold. In addition, by controlling the private market parameter, this paper is able to study the equilibrium solutions under varying relative strengths of the countervailing incentive effect to the direct effort incentive effect, including cases where one or other is dominant.

The results confirm the dependence of separating or pooling equilibria on the curvature of the reservation utility. By allowing the reservation utility to vary exogenously with a private market parameter, the solution technique of Maggi and Rodriguez-Clare (1995) enables this analysis to gain a clear insight into how the strength of the countervailing incentive affects the relationship between the information rents and the effort profiles that the principal chooses to minimise its expected procurement cost, and the ability of the principal to implement the optimal contract with a menu of linear contracts.

It must be noted that the countervailing incentive problem arises in a number of other contexts. It arises in Champsaur and Rochet's (1989) study of the interaction between two competing firms, in Laffont and Tirole's (1990) examination of bypass of a regulatory regime,

[3]Rogerson (1994) places the model more concretely: it is well known that first-best contracts can be achieved when there is (i) complete information and (ii) when all parties are risk-neutral and (iii) there is a deterministic relationship between cost and effort. Relaxing assumptions (ii) and (iii) lead to the literature of moral hazard models where the optimal incentive scheme trades off risk allocation for effort inducement (see Shavell (1979), Holmstrom (1979), Hart and Holmstrom (1987) and Mas-Colell et al, (1995 Ch. 14)). Relaxing only assumption (i) gives rise to the Laffont-Tirole analysis, where the fundamental trade-off between efficiency and rent extraction arises. The trade-off is driven by the agent's private information about its exogenously given efficiency parameter (type) and an endogenous effort variable that has desirable cost-reducing consequences, but which brings unobservable disutility to the agent.

in Biglaiser and Mezetti (1993) in the context of competing principals and in Acconcia et al (2008) in a model of vertical restraints. A more recent significant analysis of countervailing incentives is Jullien (2000), who provides a general treatment of the problem where the type-dependent individual rationality constraints give rise to countervailing incentives. While Jullien's paper is very general, this paper follows the methodology of Maggi and Rodriguez-Clare (1995).

3 Analytical Framework

The agent is characterised by two parameters: its ex ante cost $\theta \in [\theta_0, \theta_1]$ and a private market parameter, $s \in \mathbb{R}_+$, which indexes some aspect of the agent's performance in the outside private market for its services. For example, the parameter could index the degree of market power the agent exerts in the private market, or reflect the quality of the service that the agent offers. It is assumed that the private market parameter is observable by the principal, but that the cost θ is not. More precisely, the principal has a prior belief of the distribution of the agent's cost (or type) given by the common knowledge cumulative distribution function $F(\theta)$ with density $f(\theta)$ defined over the interval $[\theta_0, \theta_1]$. The distribution $F(\theta)$ is assumed to satisfy the monotone hazard rate property:

$$\frac{d}{d\theta}\frac{F(\theta)}{f(\theta)} \geq 0 \geq \frac{d}{d\theta}\frac{1-F(\theta)}{f(\theta)}$$

The principal is a Stackelberg leader who contracts with the agent for one unit of a good or service, and compensates the agent with a monetary transfer, y, over and above reimbursement of production costs. The agent's total cost of production is:

$$C = \theta - e \tag{3.1}$$

where $e \geq 0$ is the agent's effort. If the agent exerts effort level e, it decreases the monetary cost of production by one dollar and incurs a disutility of effort $\psi(e)$ measured in units of utility. It is assumed that effort is positive or zero over the range of equilibrium efforts of interest. Note that disutility only occurs when effort is strictly positive and increases with effort at an increasing rate: $\psi(0) = 0$, $\psi'(e) > 0$, and $\psi''(e) > 0$. Also $\lim_{e \to \theta} \psi(e) = +\infty$, and it is assumed that $\psi''' \approx 0$.

Total cost is observed by the principal, and by accounting convention, cost is reimbursed to the agent by the principal. Public funds are used to finance the transfers made to the agent, so the cost of the project is as perceived by the taxpayer.

An alternative specification for the agent's total cost is to introduce additive noise: $C = \theta - e + \varepsilon$, where ε is an accounting or forecast of production error with mean zero. Laffont and Tirole (1986) show that in such instances, if both the principal and agent are risk neutral then the optimal contract may still be implemented by a menu of linear contracts. This happens because the agent's incentives taken over the expectation of ε remain identical as without noise. However, if the agent is risk averse, then the linearity property no longer holds in general (see section IV and Appendix D in Laffont and Tirole (1986)).[4] Hence, to maintain an environment in which linear contracts are robust to noise, it is necessary to confine the analysis to the case where both the principal and agent are risk neutral. Then the impact of type-dependent reservation utility on the optimal contracts can be isolated.

The reservation utility of an agent of type θ is represented by $\pi(\theta, s)$, where $s \in \mathbb{R}_+$ is the private market parameter. A number of assumptions are made on this function:

1. $\pi_\theta < 0$: the reservation utility is higher for lower cost agents, as they have greater value in the private market.

2. $\pi_s \geq 0$: the reservation utility increases in the private market parameter, which is some index like quality, or product differentiation, or degree of market power.

3. $\pi_{\theta s} \leq 0$: the marginal benefit to an agent from understating their cost is greater for higher values of the private market parameter.

No assumption is made on the curvature of the reservation utility in cost type yet: in finding feasible solutions, bounds for the curvature will be derived.

In order to induce the agent to enter the contract, the principal must offer at least as much utility to the agent as it would obtain in its outside opportunity, which depends on the magnitude of the private market parameter. Following Maggi and Rodriguez-Clare

[4] Baron and Besanko (1988) show that production forecast errors lead to greater insurance (lower powered incentives) than accounting forecast errors when the agent is risk averse.

(1995), let U denote the agent's "net-utility": the utility received in its relationship with the principal in excess of its reservation utility. The individual rationality constraint is then:

$$(IR) \quad U(y(\theta)) = y(\theta) - \psi(e) - \pi(\theta, s) \geq 0, \quad \forall\theta \in \Theta, \ \& \text{ some } s \in \mathbb{R}_+ \tag{3.2}$$

Consumer benefit from production is captured by the real number $W \in \mathbb{R}_+$. Assuming that there is an excess burden of taxation, using public funds induces a distortion. Denote $\lambda > 0$ as the shadow cost of public funds. Then the net surplus to consumers is: $V = W - (1+\lambda)(y+C)$. The principal is assumed to be utilitarian, so ex post social welfare is:

$$V + U = W - (1+\lambda)(C + \psi + \pi) - \lambda U \tag{3.3}$$

where the individual rationality constraint (IR) has been used to substitute for the transfer y. The presence of the last term indicates that it is socially costly to leave rents to the agent. The principal's objective is to maximise social surplus, which involves minimising the expected rent paid to the agent. In a situation of incomplete information about the agent's cost type, the principal cannot avoid giving up rents to low cost agents as they can exploit the informational asymmetry to their advantage, as will be shown. However, before analysing that situation, it is useful to obtain the solution to the principal's problem in the complete information case as a benchmark. This gives the first best solution.

The principal's objective is to maximise social welfare, given by equation (3.3). Assuming that production is always worthwhile ($V + U > 0$, $\forall\theta$), then for each θ, the principal chooses the smallest transfer $y(\theta)$ to induce that type of agent to participate. Under complete information about cost θ, the first best condition is:

$$\psi'(e^*(\theta)) = 1, \quad \Rightarrow \quad e = e^*(\theta) \tag{3.4}$$

The interpretation of this condition is that the marginal disutility from exerting a unit of effort equals the marginal benefit - the monetary value of the reduction in cost due to that effort. Notice that the first best level of effort is invariant to the agent's type - the effort profile is flat. Hence, after reimbursing the total production cost $C^*(\theta) = \theta - e^*$, the principal transfers $y^*(\theta)$ to compensate the agent for the disutility cost of its effort and to cover its reservation utility: $y^*(\theta) = \psi(e^*) + \pi(\theta, s)$. Hence, all types of agents earn zero information rents. However, the transfer schedule is decreasing in θ. This is because a high cost agent has a smaller reservation utility, and needs to be compensated less over and above cost reimbursement for its disutility of effort than its low cost counterpart.

Under incomplete information about the agent's type, the principal still observes the agent's actual cost data, even if it does not know the agent's cost or the level of effort exerted to reduce cost. Then a feasible mechanism for the principal is to offer a net-transfer y for each level of observed cost C. However, such a mechanism is prone to perverse incentive effects. On one hand, a low cost agent may earn a rent by reporting its cost as high and completing the project at a higher (observable) total cost than in the first best case. In doing so, the agent reduces its effort to below the first-best level, thus reducing its disutility from effort. On the other hand, a high cost agent may earn a rent by reporting its cost as low and completing the project for a lower total cost than in the first best case. This time, the agent raises its effort above the first-best level, which increases its disutility from effort, but is rewarded through a higher transfer since the principal believes its opportunity cost is higher than it really is.

Hence, a feasible mechanism must now be based on the observed cost level and the private information. Fortunately, the Revelation Principle guarantees that there is no loss of generality in restricting the problem to truthful mechanisms. In such a mechanism, the principal asks that the agent make a report on its type, $\theta_\tau \in [\theta_0, \theta_1]$, and based on that report the principal transfers $y(\theta_\tau)$ to the agent, in addition to reimbursing its cost $C(\theta_\tau)$. Equivalently, given the report θ_τ, the principal reimburses production cost $C(\theta_\tau)$ and leaves information rent $U(\theta_\tau)$ for the agent. Truth-telling, or incentive compatibility, then requires:

$$(IC) \quad \arg\max_{\theta_\tau} \left\{ y(\theta_\tau) - \psi(\theta - C(\theta_\tau)) - \pi(\theta, s) \right\}, \quad \forall \theta \in \Theta \ \& \text{ some } s \in \mathbb{R}_+ \tag{3.5}$$

The necessary and sufficient conditions that the principal is constrained by in choosing the amount of effort and rent to offer to the agent in its contract to maximise social welfare are specified in the following Lemma:

Lemma 1 *(Necessary and Sufficient Conditions) Necessary and sufficient conditions for incentive compatibility are: (i)* $U_\theta(\theta) = -\psi'(e(\theta)) - \pi_\theta(\theta, s)$ *and (ii)* $e_\theta(\theta) \leq 1$.

Note that part (ii) of Lemma 1 is really a monotonicity condition on the total cost: $C_\theta(\theta) \geq 0$. This means that the effort profile may be downward or upward sloping over subintervals of the type space, so long as it is never steeper than (positive) unit slope.

The principal's problem is to induce an effort profile $e(\theta)$ and choose a utility profile $U(\theta)$ to maximise expected social welfare, while respecting individual rationality (so as to keep each type of agent in the contract) and accounting for the incentive effects that occur due to the informational asymmetry about the agent's cost type. The principal's problem can be stated formally:[5]

$$(P) \quad \max_{\{e,U\}} \int_{\theta_0}^{\theta_1} \left(W - (1+\lambda)\left(\theta - e + \psi(e(\theta)) + \pi(\theta,s)\right) - \lambda U\right) f(\theta) d\theta \tag{3.6}$$

subject to:

$$\begin{aligned} (IR) \quad U(\theta) &= y(\theta) - \psi(e) - \pi(\theta,s) \geq 0, \quad \forall \theta \in \Theta, \text{ and} \\ (IC) \quad U_\theta(\theta) &= -\psi'(e(\theta)) - \pi_\theta(\theta,s), \text{ for some } s \in \mathbb{R}_+ \end{aligned}$$

Truth-telling, or incentive compatibility is achieved by the principal through allowing agents a distribution of information rents $U(\theta)$, whose magnitude and rate of change in θ is governed by the strengths of local marginal incentives for each type of agent. When deciding which cost type to report to the principal, the agent balances the effects of two sources for personal marginal gain: (i) the direct effect that arises from reducing its effort level when reporting a higher cost, and (ii) the indirect effect through receiving a lower reservation utility when reporting a higher cost. The necessary condition of Lemma 1 for incentive compatibility bears out these two effects:

$$U_\theta(\theta) = -\underbrace{\psi'(e(\theta))}_{+} - \underbrace{\pi_\theta(\theta,s)}_{-} \tag{3.7}$$

The first term on the right hand side is the agent's marginal disutility saving on effort from over-reporting its cost type. If an agent of type θ purports to be an agent of type $\theta + d\theta$, it can reduce its effort and still meet the higher cost target: $dC = -de$. The personal disutility saving the agent makes on this marginal effort reduction is $-\psi'(e)de$. This acts to

[5] Note that this is the *relaxed* maximisation program, since the sufficient condition of Lemma 1 is omitted. The solution is checked ex post to ensure that it satisfies sufficiency.

increase the rent $U(\theta)$ accruing to the θ cost agent from reporting to be a $\theta + d\theta$ type. The second term on the right hand side is the agent's marginal reduction in reservation utility from over-reporting its cost type. Since higher cost types receive lower reservation utilities, this acts to reduce the rent $U(\theta)$ accruing to the θ cost agent. Hence, this second indirect incentive effect is called the "countervailing incentive effect". The balance of these two local incentive effects determines the slope of the utility profile at each particular value of θ.

Notice that the right hand side of the necessary condition for IC depends implicitly on two variables (holding constant the agent's type): effort, and the private market index. The dependence of the utility profile on each of these variables will be examined in turn.

First, consider the private market effect. This variable impacts the utility profile exogenously. To see how, recall that $\pi_{\theta s} \leq 0$ by assumption. Therefore, when the private market parameter s is very small, so the agent has little or no market power for example, the countervailing incentive term $\pi_\theta(\theta, s)$ is close to zero in absolute value (recall that $\pi_\theta(\theta, s) < 0$). In this case, it is likely that the direct (effort) incentive effect dominates and the utility profile will be downward sloping over all cost types of the agent. This yields the standard monotonicity constraint (or constant sign condition (CS^-) as in Guesnerie and Laffont (1984)). The CS^- condition holds because there is a systematic incentive for the agent to over-report its cost type, no matter what its actual cost type. When CS^- holds, the equilibrium contracts are fully separating - one contract is designed for each possible cost type of agent. Thus, the incentive constraint is locally downward binding. In fact, monotonicity means that the incentive and individual rationality constraints can be combined together to solve the initial value problem comprising the necessary condition in Lemma 1 and $U(\theta_1) = 0$ to yield an expression for the type θ agent's rent:

$$U(\theta) = \int_\theta^{\theta_1} \left(\psi'(e(\tilde{\theta})) + \pi_\theta(\tilde{\theta}, s)\right) d\tilde{\theta} \tag{3.8}$$

Now consider the case where the private market index is very large, so the agent has a lot of market power: then $\pi_\theta(\theta, s)$ is relatively large and negative. In this case, it is likely that the countervailing incentive effect is dominant and the utility profile will be sloping upward over all cost types of the agent. In this case, the agent has a systematic incentive to under-report its cost type, and the incentive constraint is locally upward-binding. The monotonicity constraint would be of the CS^+ type, and the initial value problem $U_\theta = -\psi'(e(\theta)) - \pi_\theta(\theta, s)$ and $U(\theta_0) = 0$ solves to yield:

$$U(\theta) = -\int_{\theta_0}^{\theta} \left(\psi'(e(\tilde{\theta})) + \pi_\theta(\tilde{\theta}, s)\right) d\tilde{\theta} \tag{3.9}$$

For intermediate values of the private market index, the sign of the left hand side of the necessary condition for IC changes. When this happens, the monotonicity requirement does not hold: the constant sign condition is violated. In general, when this happens the optimal solution entails some bunching on the subinterval of types where the constant sign condition changes.[6] However, here a special optimal control technique developed by Maggi and Rodriguez-Clare (1995) is used. This is the topic of the next section.

Before moving to the formal maximisation problem, consider the effort variable inside the necessary condition for IC. The magnitude of the direct incentive effect depends on the effort profile. Moreover, inspection of the rent equations above show that the effort profile has a cumulative effect on the rent for any given cost type in general. Hence, if the principal can control the amount of effort it induces the agent of type θ to exert, it can distort the distribution of information rents to its advantage. In fact, as the next section will describe, the principal optimally chooses an induced effort profile to limit the information rents that it has to given up to lower cost agents. It does so by trading off rent reduction through an efficiency distortion away from the first best level.

Throughout the analysis, it is assumed that the gross project value, W is always large enough that shut down of some types is never optimal. That is, the principal finds that the project is viable for all types that the agent can take. It is conceivable that this would not always be true, in which case the principal would optimally shut down some types.[7] In such cases, the net social surplus on the marginal type that the principal should shut down would be smaller than the expected rent the principal would have to pay out to achieve incentive

[6] Guesnerie and Laffont (1984) derive the full solution for cases where the constant sign condition is violated.

[7] For example, a situation where positive reservation utilities lead to optimal shut down of types is discussed in Lu's (2009) auction design problem where potential bidders have a known positive opportunity cost of bidding. He shows that the revenue-maximising auction may implement asymmetric entry across symmetric bidders.

compatibility. The principal would then shut down higher types if the direct effort incentive effect was dominant, or lower types if the countervailing incentive effect was dominant.[8]

An optimal control technique developed in Maggi and Rodriguez-Clare (1995) is used here, and the derivation closely follows theirs. In step with the usual procedure for solving incentive problems, the global second order condition is checked ex-post. Hence, the principal's problem is as before in (P) - equation (3.6). The control variable is the (induced) effort level e, the state variable is net-utility U and the costate variable is μ. Hence, the Hamiltonian is:

$$\mathcal{H} = (W - (1+\lambda)(\,\theta - e + \psi(e) + \pi) - \lambda U)\, f(\theta) - \mu(\theta)\,(\psi'(e) + \pi_\theta) \tag{3.10}$$

The point of departure here from usual incentive problems is that the issue of countervailing incentives as discussed in the previous section means that the IR constraint may bind on a subset of types in a non-trivial way. Hence, it is not possible to simply exploit the monotonicity of the utility profile to eliminate the IR constraint from the problem. To deal with this, the constraint is modeled explicitly by formulating the Lagrangian for the problem:

$$\mathcal{L} = \mathcal{H} + \tau U$$

where τ is the Lagrange multiplier for the IR constraint. Applying the Maximum Principle to this Lagrangian, as in Leonard and Long (1992), yields the following necessary conditions:

$$\psi'(e(\theta)) = 1 - \frac{1}{1+\lambda}\frac{\mu(\theta)}{f(\theta)}\psi''(e(\theta)) \tag{E}$$

$$\mu_\theta = \lambda f(\theta) - \tau(\theta) \tag{C}$$

$$U_\theta(\theta) = -\psi'(e(\theta)) - \pi_\theta(\theta, s) \tag{S}$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = U(\theta) \geq 0, \quad \tau(\theta) \geq 0, \quad U(\theta)\tau(\theta) = 0 \tag{CS}$$

$$\mu(\theta_0) \leq 0, \quad \mu(\theta_0)U(\theta_0) = 0 \text{ and } \mu(\theta_1) \geq 0, \quad \mu(\theta_1)U(\theta_1) = 0 \tag{TV}$$

[8]Technically, it can be shown that if there exists a $\theta^* \in (\theta_0, \theta_1)$ defined by $(W - (1+\lambda)(\theta^* - e^* + \psi(e(\theta^*)) + \pi(\theta^*, s)))f(\theta^*) = \lambda(\psi'(e(\theta^*)) + \pi_\theta(\theta^*, s))F(\theta^*)$, then cost types greater than θ^* should be shut down whenever the direct incentive effort incentive dominates and cost types lower than θ^* should be shut down whenever the countervailing incentive effect dominates.

The transversality conditions nest the possibilities for the endpoints of the solution profiles. To see this, take the case where the direct (effort) incentive effect dominates in the IC. Then low cost agents will inevitably earn information rents by over-reporting their cost. To ensure incentive compatibility, the principal must leave rents to the low cost agent, but will reduce the utility profile over the type space to ensure that the highest cost type receives no rent, where the IR constraint will be binding. To *minimise* the expected information rent, the principal can distort the effort required for higher cost types lower than first best, which prevents the gain from high cost types saving on effort reduction. So at some $\hat{\theta}$, the costate variable $\mu^*(\hat{\theta})$ is the imputed value to the principal of retaining the rent for types greater than $\hat{\theta}$, and should therefore be positive. Hence, in such cases, the shadow price of rent to the principal at the highest cost type is positive, $\mu(\theta_1) > 0$, and the principal leaves that cost type with no rent: $U(\theta_1) = 0$. For the opposite case where the countervailing incentive effect dominates, the incentives work in the opposite direction. The value of the shadow price of rent to the principal is greatest at the lowest cost type because the principal receives no benefit from incentivising that cost type to report its true type. Hence $\mu(\theta_0) > 0$, and $U(\theta_0) = 0$ in that case.[9]

Since $\mathcal{H}(U, e, \mu, \theta)$ is concave in U, if the configuration $(U(\theta), e(\theta), \mu(\theta), \tau(\theta))$ satisfies equations (E)-(TV) it also satisfies the sufficient conditions for an optimum. Let $e^{\mu}(\theta)$ denote the value of e that maximises $\mathcal{H}$ given μ and θ, defined by equation (E). Let $\nu(\theta, s)$ be the solution to the state differential equation (S): $U_\theta = -\psi'(e^{\nu}(\theta)) - \pi_\theta(\theta, s)$. Then $\nu(\theta, s)$ is the value of the costate variable such that $U_\theta(\theta) = 0$. It is clear that $\mu(\theta) = \nu(\theta, s)$ on any interval where the IR constraint is binding (and in that case, the principal should optimally pin down the net-utility to zero for that interval).

The $\nu(\theta, s)$ solution has a neat graphical interpretation: it is the locus of points in (μ, θ) space where $U_\theta(\theta) = 0$. Figure 1 illustrates this for a particular solution of $\nu(\theta, s)$ indicated by the heavy trace. Evidently, in the same space the region above this locus corresponds to $U_\theta(\theta) < 0$, and everywhere below has $U_\theta(\theta) > 0$. Recall that $\mu(\theta) = \lambda F(\theta)$ is the optimal

[9] The economic interpretation of the costate variables are not discussed in Maggi and Rodriguez-Clare (1995), but are important to understanding the problem. This generalised "shadow price" interpretation of the costate variable is derived in Leonard (1987) and discussed intuitively in Leonard and Long (1992).

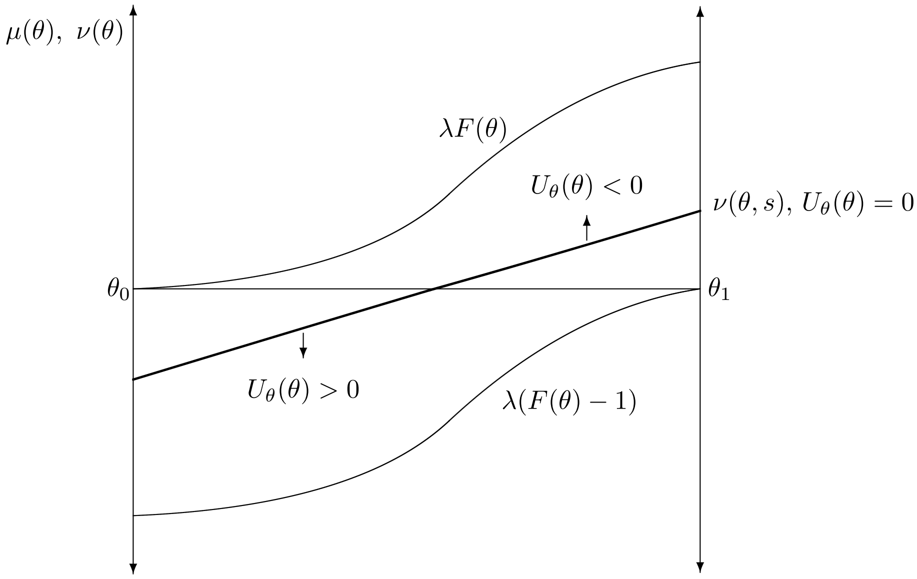

Figure 1: An Example Solution for $\nu(\theta, s)$

costate trajectory whenever $U_\theta(\theta) < 0$ and $\mu(\theta) = \lambda(F(\theta)-1)$ is the optimal costate solution whenever $U_\theta(\theta) > 0$. These are indicated in Figure 1 by the light traces. Hence, the optimal costate trajectory follows $\lambda F(\theta)$ whenever it lies below $\nu(\theta, s)$, and then becomes the $\nu(\theta, s)$ solution itself whenever $\lambda F(\theta)$ is above. Likewise, whenever $\lambda(F(\theta) - 1)$ lies above $\nu(\theta, s)$ it represents the optimal costate solution, and follows $\nu(\theta, s)$ when it is above. With this in mind, the conjectured solution is defined over a partition of the type space. In particular, let $\Theta = \Theta_I \cup \Theta_{II} \cup \Theta_{III}$ where the partitions are defined by $\Theta_I := \{\theta : \lambda F(\theta) \leq \nu(\theta, s)\}$, $\Theta_{II} := \{\theta : \lambda(F(\theta) - 1) \leq \nu(\theta, s) \leq \lambda F(\theta)\}$, $\Theta_{III} := \{\theta : \nu(\theta, s) \leq \lambda(F(\theta) - 1)\}$. Then the conjectured solution is:

$$\mu^*(\theta) = \begin{cases} \lambda F(\theta) & \text{for all} \quad \theta \in \Theta_I \\ \nu(\theta, s) & \text{for all} \quad \theta \in \Theta_{II} \\ \lambda(F(\theta) - 1) & \text{for all} \quad \theta \in \Theta_{III} \end{cases} \tag{3.11}$$

Exactly which profile the optimal costate variable takes depends on the position of the $\nu(\theta, s)$ solution, which in turn depends on the private market index.

Inspection of the (point-wise) first order condition for effort in equation (E) and the conjectured costate solution reveals an important insight into the principal's equilibrium contracts. The distortion in the effort for incentive compatibility depends on the magnitude of the costate variable at each cost type. As such, if there is a change in the slope of the costate solution across partitions, then effort is not distorted in a uniform direction. This will be manifest as a non-convexity, or wrinkle in the net-transfer rule - which precludes the ability of the principal to use a menu of linear contracts. So in the next section, a careful study is made of the position and shape of the $\nu(\theta, s)$ solution for different values of the private market parameter. Note also the dependence of the slope of $\nu(\theta, s)$ on the curvature of the reservation utility through the state equation (S). Whether the reservation utility is concave or convex will dictate the feasibility of implementation with a menu of linear contracts.

In the next section, the conditions on the curvature of the reservation utility that preserve the linearity property of optimal procurement contracts are derived. Since the strength of countervailing incentives depends on the magnitude of the private market parameter, solutions corresponding to various levels of s are examined. An equilibrium will be denoted $\Big(U^*(\theta), e^*(\theta), \mu^*(\theta) \mid_{s\in\mathbb{R}_+}\Big)$, as the solution to the principal's problem outlined in equations (E), (S), (C), (CS) and (TV) using the conjectured costate solution (3.11) for a particular value of the private market parameter, $s \in \mathbb{R}_+$.

4 Implementation with a Linear Menu

This section derives the net-transfer rule that implements the solution to the principal's problem, $\Big(U^*(\theta), e^*(\theta), \mu^*(\theta) \mid_{s\in\mathbb{R}_+}\Big)$ for various values of the private market parameter. Most importantly, the conditions on the convexity of the agent's reservation utility are obtained that allow the solution to be implemented with a menu of linear contracts. This linearity property of optimal procurement contracts holds whenever the net-transfer rule is decreasing, convex and continuously differentiable.[10] This section establishes the conditions required for the net-transfer rule to be decreasing and convex. The next section discusses continuous

[10]A function that is continuously differentiable is one whose derivative is continuous. This simply rules out kinks in the net-transfer rule. If the net-transfer rule has a kink, then the left and right limits of the derivative at the kink are not equal, so the derivative of the rule is not continuous.

differentiability.

From Lemma 1, in a solution to the principal's problem the total cost $C(.)$ is a strictly increasing function of θ. Hence, it can be inverted so that $\theta = \theta^*(C)$. Rewriting equation (3.2) then yields:

$$Y^*(C) = \psi(e^*(\theta^*(C))) + \pi(\theta^*(C), s) + U^*(\theta^*(C)), \quad s \in \mathbb{R}_+ \tag{4.1}$$

To establish whether the net-transfer rule is decreasing and convex, differentiate equation (4.1):

$$\frac{dY^*}{dC} = -\psi'(e^*) \quad \text{and} \quad \frac{d^2Y^*}{dC^2} = -\psi''(e^*)\frac{e^*_\theta}{1 - e^*_\theta} \tag{4.2}$$

where the necessary condition for incentive compatibility has been used in the left hand equation. Since the disutility of effort function is assumed to be increasing and convex, the cost-reimbursement rule is always decreasing. However, global convexity requires the optimal effort profile to be decreasing: $e^*_\theta(\theta) \leq 0$. Laffont and Tirole's (1993) work shows that a sufficient condition for implementability with a menu of linear contracts is for the monotone hazard rate assumption to hold. This is equivalent to requiring that the effort profile be decreasing in cost type.[11] Recall that this is a stronger condition than what is required for sufficiency for an equilibrium, as outlined out in part (ii) of Lemma 1: $e^*_\theta \leq 1$. The Laffont-Tirole linearity property is gathered in Proposition 1 below:

Proposition 1 *(Linearity Property) If a solution to the principal's problem (P) given by* $\left(U^*(\theta), e^*(\theta), \mu^*(\theta) \mid_{s\in\mathbb{R}_+}\right)$ *entails an effort profile that is downward sloping,* $e^*_\theta(\theta) \leq 0$*, then the optimal contract can be implemented with a menu of linear contracts.*

The implication is that the requirements for implementation by a linear menu are stronger than for an equilibrium to exist: the net-transfer rule may in fact exhibit non-convexities, and yet still support a solution. Moreover, the slope of the equilibrium effort profile for a particular value of the private market parameter is inextricably tied to the curvature of the reservation utility, as will be discussed in detail below.

[11] Laffont and Tirole's (1986) analysis with normalised outside opportunity is equivalent to the case here where $\pi_\theta = 0$ for all θ. Then $\mu(\theta) = \lambda F(\theta)$ and the effort profile given implicitly by equation (E) is always decreasing provided $\frac{d}{d\theta}(\frac{F}{f}) \geq 0$.

Notwithstanding, if the effort profile is strictly decreasing, then equations (4.2) imply that the net-transfer rule in equation (4.1) is decreasing and convex. Hence, it can be replaced with a menu of its tangents - a menu of contracts that are linear in cost over-runs or under-runs. This is illustrated in the left hand side of Figure 2. The dashed tangent lines to the net-transfer rule indicate linear contracts designed for two possible cost types, θ_L and θ_H. If the agent is of type θ_L, it will choose to take the steeper linear contract, exert effort level e_L^* and land the project for C_L. The principal then reimburses the agent for total cost C_L and transfers $Y(C_L)$. Similarly for the higher cost agent. Note that neither agent has incentive to take the other's contract. This is because the net-transfer rule embodies incentive compatibility. Now consider what happens if the rule has a non-convexity, as in

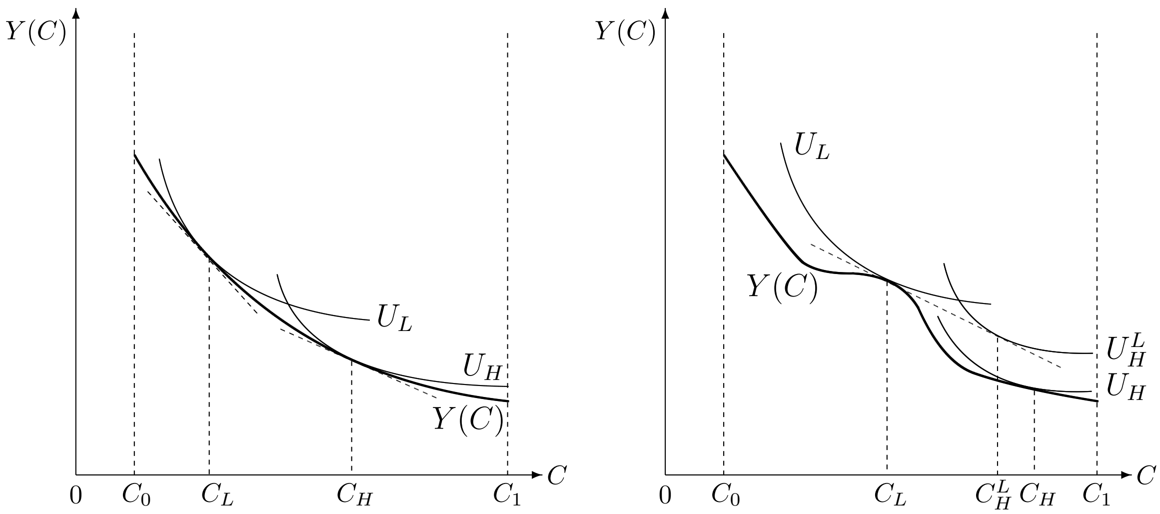

Figure 2: Implementation with a Menu of Linear Contracts

the right hand side of Figure 2.[12] The higher cost agent clearly has incentive to exert more effort and report to be the lower cost agent that has experienced a cost over-run. The larger transfer from the principal more than compensates the agent for the extra effort it had to exert. Hence, incentive compatibility is violated. Importantly, the net-transfer rule (in bold) is a solution. However, the pertinent point here is simply that it cannot be implemented with a menu of linear contracts.

[12]The right hand side of Figure 2 is drawn to be continuously differentiable. However, if there was a kink in the rule at C_L, a similar problem would exist.

If the net-transfer rule is decreasing, convex and continuously differentiable, the menu of linear contracts can be represented as:

$$y(\hat{\theta}, C) = y^*(\hat{\theta}) - \psi'(e^*(\hat{\theta}))(C - C^*(\hat{\theta})), \quad \forall \hat{\theta} \in \Theta \tag{4.3}$$

so a linear contract is offered to each cost type of agent. Note that $y^*(\hat{\theta})$ is simply the net-transfer that the agent of type $\hat{\theta}$ would receive if facing the optimal menu. Also, since $\psi'(.) > 0$, the coefficient of cost over-runs represents the power of the incentive contract, which is higher for lower cost types. For the lowest cost type of agent, $\psi'(e^*(\theta_0) = 1$, and the menu prescribes a fixed price contract (the highest powered incentives).[13]

To check that an agent of cost type $\hat{\theta}$ is induced to report its true cost type, note that the agent's problem is to simultaneously choose a cost type to report, $\hat{\theta}$, and a level of effort to exert to ensure the project is landed for its report cost: $\hat{C} = \hat{\theta} - e$:

$$\max_{\{\hat{\theta}, e\}} \left[\psi(e^*(\hat{\theta})) + \pi(\hat{\theta}, s) + U^*(\hat{\theta}) - \psi'(e^*(\hat{\theta}))((\theta - e) - (\hat{\theta} - e^*(\hat{\theta})) - \psi(e)\right] \tag{4.4}$$

where the first three terms in the maximisation comprise the transfer $y^*(\hat{\theta})$. The first order conditions for the agent's problem are:

$$\begin{aligned} \psi' e^*_\theta + \pi_\theta - \psi' - \pi_\theta - \psi'(e^*_\theta - 1) - \psi''(\theta - e - (\hat{\theta} - e(^*\hat{\theta})))e^*_\theta &= 0 \\ \psi'(e^*(\hat{\theta})) - \psi'(e) &= 0 \end{aligned}$$

The second of these first order conditions shows that the menu of linear contracts induces the same level of effort from the agent as the optimal menu induces: $e^*(\hat{\theta}) = e$. The first condition then shows that the agent reports the truth: $\hat{\theta} = \theta$.

Now that the conditions on the effort profile for the linearity property to hold have been established in Proposition 1, and it has been shown that a linear menu can induce truth-telling, the next step is to establish the conditions on the convexity of the agent's reservation utility that ensure the solution entails an effort profile that is downward sloping. This is undertaken in the next section.

[13] It is also worth pointing out that the agent experiences no uncertainty in this model: in equilibrium there is no cost under-run or over-run.

4.1 Convexity of the Reservation Utility

This section constructs the regularity conditions on the convexity of the reservation utility that are required for the linearity property to be extended to this generalised procurement environment. Recall that the linearity property holds when the agent's reservation utility is normalised to zero, provided the effort profile is always downward sloping. However, now the equilibrium effort profile in general depends on the magnitude of the private market parameter, and the convexity of the reservation utility. This is because the private market parameter influences the position of the $\nu(\theta, s)$ solution in $[\mu^*, \theta]$ space, and the convexity of the reservation utility affects its slope, as seen below in equation (4.5). Moreover, through equation (E), the size and direction of the distortion of effort away from the first best level, and the slope of the effort profile both depend on the position and slope of $\nu(\theta, s)$ whenever countervailing incentives bind in the solution. Then to proceed, the slope of the $\nu(\theta, s)$ solution and the impact of s on its position are analysed.

To obtain the shape of the $\nu(\theta, s)$ costate solution, differentiate equation (S) at the equilibrium solution (where $U_\theta(\theta) = 0$):

$$\nu_\theta(\theta, s) = \left(-e_\theta^\nu - \frac{\pi_{\theta\theta}}{\psi''} \right) / e_\mu^\nu \tag{4.5}$$

Returning to the first order conditions from the Maximum Principle, condition (CS) implies that $U(\theta) = 0$ whenever $\tau(\theta) \geq 0$. Hence equation (C) gives $\tau(\theta) = \lambda f(\theta) - \nu_\theta(\theta) \geq 0$.[14] Thus, for an interval of types over which the IR constraint binds to be an optimum, the slope of the costate profile must not be *too* steep: $\nu_\theta(\theta) \leq \lambda f(\theta)$, which is the analogous condition to Maggi and Rodriguez-Clare (1995).

Turning now to the linearity property requirement that the slope of the effort profile be always downward sloping, differentiating equation (E) yields:[15]

$$\frac{de^*}{d\theta} = -\frac{1}{1+\lambda} \frac{d}{d\theta} \left(\frac{\mu^*}{f} \right) \tag{4.6}$$

From the definition of the conjectured costate solution in equation (3.11), it follows from (4.6) that the effort profile is downward sloping on partitions Θ_I and Θ_{III} since $\frac{d}{d\theta}\left(\frac{F}{f}\right) > 0$

[14] When the IR constraint is not binding, $\mu_\theta(\theta) = \lambda f(\theta)$ and then the transversality conditions (TV) give the usual cases of always downward binding or always upward binding incentive compatibility.

[15] Formally, $\psi'' \frac{de^*}{d\theta} = -\frac{\psi''}{1+\lambda} \frac{d}{d\theta} \left(\frac{\mu^*}{f} \right) + O(\psi''')$, where $O(\psi''') \approx 0$.

by assumption. On partition Θ_{II}, equation (4.6) implies that the requirement is $\frac{d}{d\theta}\left(\frac{\nu}{f}\right) > 0$. Lemma 2 states the convexity requirements on the reservation utility that meet these conditions:

Lemma 2 *Provided that* $\pi_{\theta\theta}(\theta, s) \in [0, \hat{\pi}_{\theta\theta}]$ *then: (i)* $\nu_\theta(\theta, s) \leq \lambda f(\theta)$*, and (ii)* $e^*_\theta(\theta) \leq 0$*, where* $\hat{\pi}_{\theta\theta} := \psi'' \frac{\lambda}{1+\lambda} \frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)$.

Under the conditions specified in the Lemma, the $\nu(\theta, s)$ solution never has a slope greater then the slopes of the costate variable on the other two partitions. This implies that to maximise expected welfare, the principal need not distort the effort profile away from the first best as much over the interval on which the individual rationality constraint is binding. This is because the principal optimally reduces the agent's induced effort to offset the countervailing incentive at the exact rate required to keep each type of agent on its reservation utility.

Having established the convexity requirements for the shape of the $\nu(\theta, s)$ costate solution, now the position of the solution with respect to the private market parameter will be established. Note that for very low values of s, say very low quality, the reservation utility of the agent is essentially flat: the standard direct effort incentive should dominate. Conversely, when quality is very high, the agent's outside opportunity is high, so the countervailing incentive effect should be dominating. This suggests that for higher values of the private market parameter, the $\nu(\theta, s)$ solution should shift downward in $[\mu^*, \theta]$ space. The following Lemma formalises this intuition:

Lemma 3 *The* $\nu(\theta, s)$ *solution shifts down as the value of the private market parameter increases:* $\nu_s = -\frac{\pi_{\theta s}}{e_\nu \psi''} < 0$.

Inspection of Figure 3 reveals that where the $\nu(\theta, s)$ solution intersects either $\mu^* = \lambda F(\theta)$ or $\mu^* = \lambda(F(\theta) - 1)$ defines the boundary of partitions I and II or II and III respectively. For values of the private market parameter like s_L, s_3 or s_H, there is no intersection of partitions. Then the critical cost type $\hat{\theta}$ is defined as the cost type on the boundary of these partitions: $\hat{\theta} \in \{\Theta_I \cap \Theta_{II}, \Theta_{II} \cap \Theta_{III}, \phi\}$, where ϕ represents no partition.

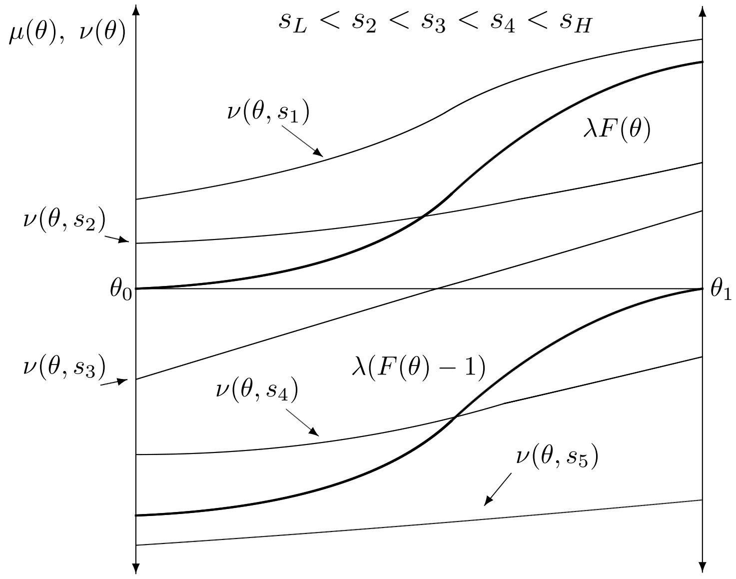

Figure 3: Costate Solutions for Various s-values

The equilibrium effort and information rent solutions to the principal's problem (P) for the private market parameter values from Figure 3 when the curvature of the reservation utility is $\pi_{\theta\theta}(\theta, s) \in [0, \hat{\pi}_{\theta\theta}]$ are listed in Table 1.

Table 1. Solution Profiles for: $\pi_{\theta\theta} \in [0, \hat{\pi}_{\theta\theta}]$

Solution Profile	Effort Profile	Rent Profile
Type L: $\Theta = \Theta_I$ Critical $\hat{\theta} = \phi$	$e^L(\theta) \leq e^{fb}$	$U^*(\theta) = \int_{\theta}^{\theta_1} (\psi'(e) + \pi_\theta) d\tilde{\theta}$
Type s_2: $\Theta = \Theta_I \cup \Theta_{II}$ Critical $\hat{\theta} = \Theta_I \cap \Theta_{II}$	$e^{fb} \geq e^2(\theta) \geq e^L(\theta)$	$U^*(\theta) = \int_{\theta}^{\hat{\theta}} (\psi'(e) + \pi_\theta) d\tilde{\theta}$ for $\theta \leq \hat{\theta}$ and 0 otherwise
Type s_3: $\Theta = \Theta_{II}$ $\hat{\theta} = \{\theta \mid \nu(\theta, s) = 0\}$	$e^3(\theta) \geq e^{fb}$ for $\theta \leq \hat{\theta}$, otherwise $e^{fb} \geq e^3(\theta)$	$U^*(\theta) = 0$ for all θ
Type s_4: $\Theta = \Theta_{II} \cup \Theta_{III}$ Critical $\hat{\theta} = \Theta_{II} \cap \Theta_{III}$	$e^H(\theta) \geq e^4(\theta) \geq e^{fb}$	$U^*(\theta) = -\int_{\hat{\theta}}^{\theta} (\psi'(e) + \pi_\theta) d\tilde{\theta}$ for $\theta \geq \hat{\theta}$ and 0 otherwise
Type H: $\Theta = \Theta_{III}$ Critical $\hat{\theta} = \phi$	$e^H(\theta) \geq e^{fb}$	$U^*(\theta) = -\int_{\theta_0}^{\theta} (\psi'(e) + \pi_\theta) d\tilde{\theta}$

Solution types L and H correspond to the scenarios where either the direct effort incentive is dominant (type L), or the countervailing incentive is dominant (type H). As depicted in the light traces of the left hand panel in Figure 4, for the type L solution the principal optimally distorts the effort levels downward from first best in order to limit the information rent that it must give up to low cost types of the agent to prevent them from over-reporting their cost. On the other hand, the right hand panel of Figure 4 shows that for the Type H solution, the principal optimally distorts the effort levels upward from first best in order to limit the reservation utility it must pay out to higher cost agents to prevent them from under-reporting their cost. The interesting cases occur for intermediate levels of the private market parameter, where the balance of the direct effort and countervailing incentives changes over the type space.

The heavy traces in the left hand side of Figure 4 depicts the case where countervailing incentives becomes dominant on a subset of the higher cost type agents. Then the conjectured costate solution of equation (3.11) forces the state variable, net-utility, to be governed along a path of zero net-utility corresponding to $\nu(\theta, s)$ for those higher cost types. Since the values of $\nu(\theta, s)$ over the subset of higher cost types is still positive, but smaller than $\lambda F(\theta)$ then equation (E), effort is distorted to a lessor degree than under the fully dominant direct effort case. A similar effect occurs for higher values of the private market parameter like s_4 in the far right panel of Figure 4. This statement is made precise in the following Corollary:

Corollary 1 *Suppose* $\pi_{\theta\theta}(\theta, s) \in [0, \hat{\pi}_{\theta\theta}]$. *Then* $\forall \theta \in \Theta_{II}$: $e^L(\theta) \geq e^\nu(\theta) \geq e^H(\theta)$

The corresponding utility profiles for each case, as illustrated in Figure 4 suggest two things. First, that the critical value of $\hat{\theta}$ (locally) decreases as the private market parameter increases.[16] Second, that the value of the utility profile decreases for all cost types less than $\hat{\theta}$ on partition I, and increases for all cost types greater than $\hat{\theta}$ on partition III. The details are in the proof of the following Corollary:

Corollary 2 *Suppose* $\pi_{\theta\theta}(\theta, s) \in [0, \hat{\pi}_{\theta\theta}]$. *Then:* $\frac{\partial U(\tilde{\theta})}{\partial s} \leq 0$ *for* $\tilde{\theta} \in \Theta_I$, $\frac{\partial U(\tilde{\theta})}{\partial s} = 0$ *for* $\tilde{\theta} \in \Theta_{II}$ *and* $\frac{\partial U(\tilde{\theta})}{\partial s} \geq 0$ *for* $\tilde{\theta} \in \Theta_{III}$.

Figure 4 sheds light on the reasoning for the conjectured costate variable. In the top left hand side panel, the $\nu(\theta, s)$ solution intersects $\mu(\theta) = \lambda F(\theta)$ at $\hat{\theta}$. Remembering that in

[16] This is not a global relationship, since there is a discontinuity defined for values of the private market parameter like s_3.

the region below $\nu(\theta, s)$ it must be that $U_\theta(\theta) < 0$, then $\mu(\theta) = \lambda F(\theta)$ is optimal and the induced effort profile on $\Theta_I = [\theta_0, \hat{\theta}]$ is given implicitly by $\psi'(e^L) = 1 - \frac{\lambda}{1+\lambda}\frac{F}{f}\psi''(e^L)$.

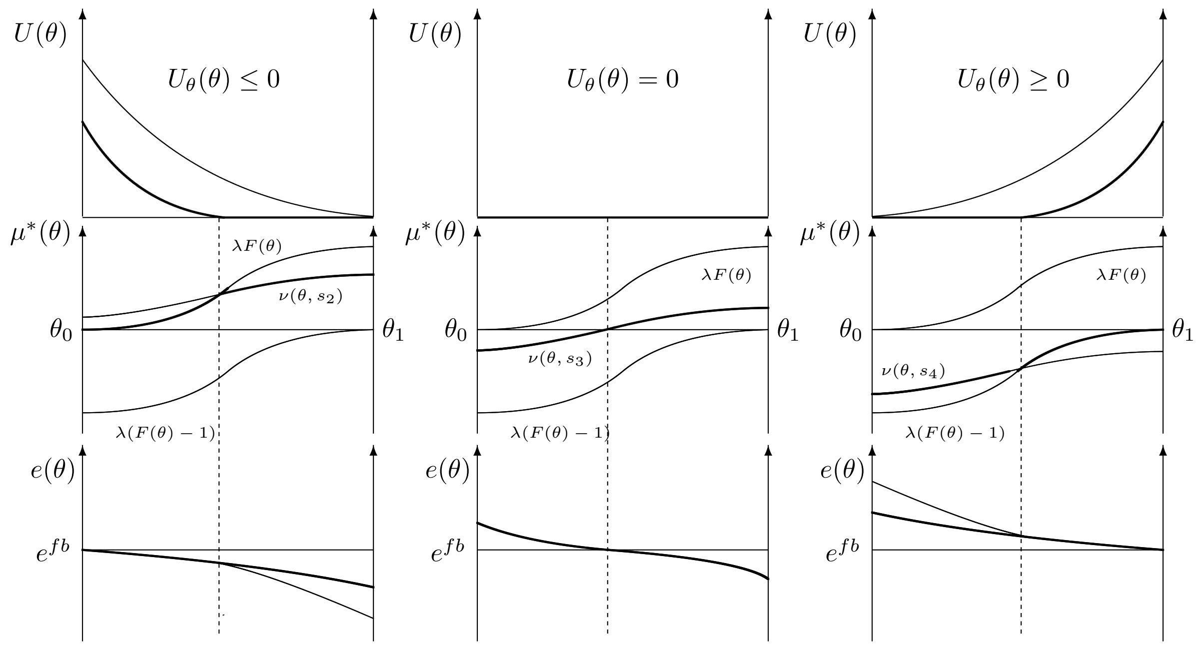

Figure 4: Utility, Costate & Effort Profiles – Convex $\pi(\theta, s)$

Moving along the $\lambda F(\theta)$ curve from the left, $\mu(\theta)$ is increasing - the value of relaxing the IC and thereby reducing the information rent left to higher-cost types impacts positively on the principal's objective function (since information rents are socially costly), so that all higher types should receive lower rent. Moving through the intersection of $\lambda F(\theta)$ with $\nu(\theta, s)$ leads to a region where $U_\theta(\theta) > 0$ so that $\lambda F(\theta)$ is no longer optimal. Pursuing that trajectory would lower the expected value of the principal's objective function, because in that region $\tau(\theta) > 0$ and the IR constraint binds, so $U(\theta) = 0$ in the solution to (P). Instead, the costate solution optimally tracks $\nu(\theta, s)$, so that $U_\theta(\theta) = 0$. Also, by requiring $U(\hat{\theta}) = 0$, it follows that the optimal costate variable satisfies $U_\theta(\theta) \leq 0$ ($\mu_\theta(\theta) = \lambda f(\theta)$) and $U(\hat{\theta}) = 0$ from the left, and $U_\theta(\theta) = 0$ with $U(\hat{\theta}) = 0$ from the right. The induced effort profile to the right of $\hat{\theta}$ is given implicitly by $\psi'(e^\nu) = 1 - \frac{1}{1+\lambda}\frac{\nu}{f}\psi''(e^\nu)$. Since $\nu(\theta, s) < \lambda F(\theta)$, the effort distortion is less than it would be without the countervailing incentive effect, and there is a kink in the induced effort profile at $\hat{\theta}$, as shown in the middle left hand panel.

If the value of the private market parameter is consistent with Partition III, the middle panel of Figure 4 shows that there is no part of the optimal solution for $U_\theta(\theta) < 0$ ($\mu(\theta) = \lambda F(\theta)$) that intersects with the region below $\nu(\theta, s)$, where $U_\theta(\theta) < 0$, and therefore it cannot form part of the solution. Similarly, no part of $\mu(\theta) = \lambda(F(\theta) - 1)$ intersects with the region above $\nu(\theta, s)$ where $U_\theta(\theta) > 0$, hence it cannot form part of the solution. In fact it follows from the conjectured solution that the costate variable in this cases is $\mu^*(\theta) = \nu(\theta, s)$ over the entire type space. This means that the IR constraint binds for all types, and no agent type receives information rent, as depicted at the top of the middle panel. All types receive compensation for their reported cost and their disutility of effort exerted, plus, they receive a transfer equal to the profit they would have made in the private market.

Since the IC binds on each type, for all cases discussed so far the equilibrium is fully separating. This is borne out in the middle of Figure 4, where it is seen that the induced effort profile is still downward-sloping - a result due to the fact that $\nu(\theta, s)$ is non-decreasing. Importantly, whenever $\nu(\theta) > 0$, the induced effort profile is distorted downward from first-best to relieve the principal from leaving costly information rent to low-cost types. On the other hand, whenever $\nu(\theta) < 0$, the effort profile is distorted upward so as to mitigate the costly information rents left to high-cost types. Of course when $\nu(\theta) = 0$, the first best is effort is induced for that particular type where it occurs.

5 The Extended Linearity Property

Having established the convexity requirements on the agent's reservation utility to ensure that the equilibrium effort profiles are downward sloping for all values of the private market parameter, this section examines the properties of the corresponding net-transfer rules. In particular, it remains to be seen whether the net-transfer rules are continuously differentiable: if they are not, then implementation with a menu of linear contracts is not permissible. Recall that the net-transfer rule is:

$$Y^*(C) = \psi(e^*(\theta^*(C))) + \pi(\theta^*(C), s) + U^*(\theta^*(C)), \quad s \in \mathbb{R}_+ \tag{5.1}$$

Where the countervailing incentive is strong enough that the solution is defined over two partitions (where, for example, quality is greater enough that higher cost agents begin to have incentive to under-report their cost), the solution $\left(U^*(\theta), e^*(\theta), \mu^*(\theta) \mid_{s\in\mathbb{R}_+}\right)$ is "patched" across partitions. At the borders of the partition, there is a kink in the costate profile and therefore in the effort profile, as seen for example in Figure 4 in the leftmost and rightmost cases. At first glance, this non-differentiability in the effort profile could translate into a non-convexity in the net-transfer rule. If it does, then the linearity property is void. However, Lemma 4 shows that even patched solutions give rise to decreasing and convex net-transfer functions.

Lemma 4 *If $\pi_{\theta\theta}(\theta, s) \in [0, \hat{\pi}_{\theta\theta}]$, the net-transfer rule is continuously differentiable, decreasing and convex in θ for all s.*

Intuitively, for any value of the private market parameter, s, at the border of a patched solution the slope of the utility profile approaches zero. This reflects the rate at which is optimal to trade-off observed total cost levels for transfers when the type crosses from (say) region I into II. This may seem surprising in view of the non-differentiability in the effort and costate profiles on the border of partitions. However, the non-differentiability only translates into a discontinuity in the *convexity* of the net-transfer rule: while the rule itself is smooth and continuous, its curvature undergoes a change on the border of a patched solution. The next Lemma formalises this result.

Lemma 5 *When the IR constraint is binding for a subset of agent cost types, the net-transfer rule is less convex than for subsets where it is relaxed.*

The implication of Lemmata 4 and 5 are shown in Figure 5. As the private market parameter increases so that the countervailing effect begins to bind on higher cost types, the new net-transfer function "peels off" from the original function and lies below the hypothetical net-transfer rule. Intuitively, the reason is that since the principal does not distort the effort profile downward as much for higher cost types when the IR binds, those higher cost types must exert more effort, closer to the first-best level. The higher effort requirement for higher types increases their marginal disutility from effort, making the slope of the net-transfer function steeper. Any types along the patched portion of the curve earn zero net-utility, however the effort schedule is still downward sloping.

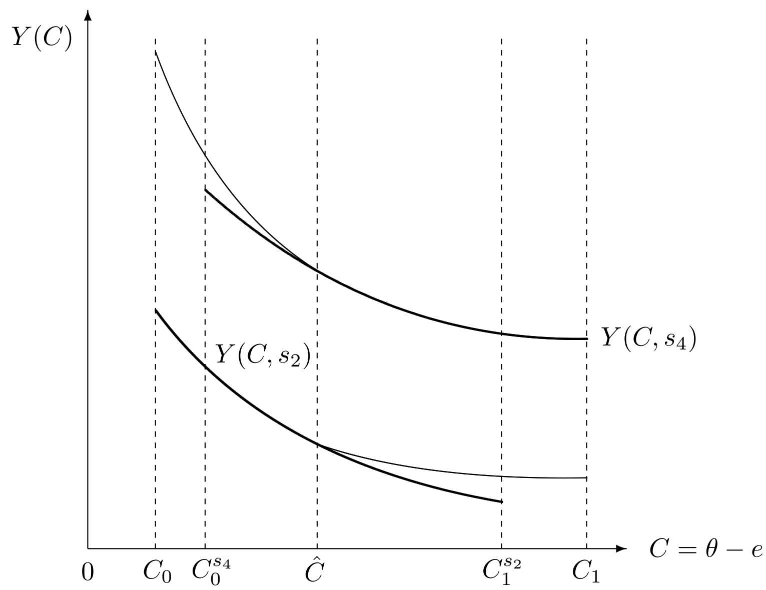

Figure 5: Net-Transfer Schedules

For high enough values of the private market parameter where countervailing incentive binds on lower types, a similar effect occurs. Now the optimal effort profile entails a smaller distortion above the first best effort than when countervailing incentives would bind on all cost types. Hence lower cost types do not exert as much effort, and the marginal disutility from effort is therefore smaller. Consequently, the new net-transfer function peels off from the original function and again lies below the hypothetical net-transfer rule.

For values of the private market parameter like s_3, where the balance of the direct effort and countervailing incentives result in a flat utility profile, the net-transfer rule is simply:

$$Y^*(C) = \psi(e^\nu(\theta^\nu(C))) + \pi(\theta^\nu(C), s_3)$$

In this case, the slope of the net-transfer rule is determined by the shape of the reservation utility, since from the necessary condition for IC, $-\psi'(e^\nu(\theta)) = \pi_\theta(\theta, s_3)$, since the utility profile is flat: $U_\theta(\theta) = 0$ over the entire cost type interval.

Lemmata 4 and 5 and the descriptive reasoning above lead to the conclusion that the net-transfer rule is decreasing and convex for all values of the private market parameter, provided that the reservation utility meets Lemma 2's convexity requirement. In such cases, it can always be replaced by the family of its tangents. This is the key result of this analysis, which generalises Laffont and Tirole's (1986) linearity property, as captured in Proposition 2:

Proposition 2 *(Extended Linearity Property) If the curvature of the agent's reservation utility is in the interval* $\pi_{\theta\theta}(\theta, s) \in [0, \hat{\pi}_{\theta\theta}]$*, then a solution to the principal's problem (P) given by* $\left(U^*(\theta), e^*(\theta), \mu^*(\theta) \mid_{s \in \mathbb{R}_+}\right)$ *can be implemented with a menu of linear contracts.*

The result implies that the class of environments in which it is convenient to work with linear incentive schemes is necessarily restricted to situations where the agent's outside opportunity is convex in cost type.

Laffont and Tirole's precise hypothesis for the linearity property to hold specified that the monotone hazard rate assumption should hold. Earlier it was argued that this is equivalent to requiring that the effort profile be downward sloping. That equivalence lead to the focus on the optimal costate, effort and rent profiles for different values of the private market parameter of the previous section to derive Proposition 2. Interestingly however, a counterpart to their hypothesis can be obtained in this generalised procurement environment that has the same substantive meaning:

Corollary 3 *A solution to the principal's problem (P) given by* $\left(U^*(\theta), e^*(\theta), \mu^*(\theta) \mid_{s \in \mathbb{R}_+}\right)$ *can be implemented with a menu of linear contracts whenever:* $\frac{d}{d\theta}\left(\frac{\mu^*(\theta}{f(\theta)}\right) \geq 0.$

So the extended linearity property amounts to a monotone restriction on the generalised costate variable, $\mu^*(\theta)$ of equation (3.11). The conjectured costate variable was in turn a response to the generalised procurement environment of type dependent reservation utilities.

To complete the analysis, attention is finally turned to the case where the reservation utility is concave. While the extended linearity property of Proposition 2 show that implementation with a menu of linear contracts will not be feasible if the reservation utility is concave, it is still possible to obtain an optimal menu, as the next section briefly illustrates.

5.1 Concavity of the Reservation Utility

When the reservation utility is concave, it may still be possible to support a solution $\left(U^*(\theta), e^*(\theta), \mu^*(\theta) \mid_{s \in \mathbb{R}_+}\right)$ with a net-transfer rule, however the rule necessarily has a non-convexity. As a result, it cannot be implemented with a menu of linear contracts. Figure

6 shows the solution profiles for such a case. As can be seen, the downward sloping $\nu(\theta, s)$

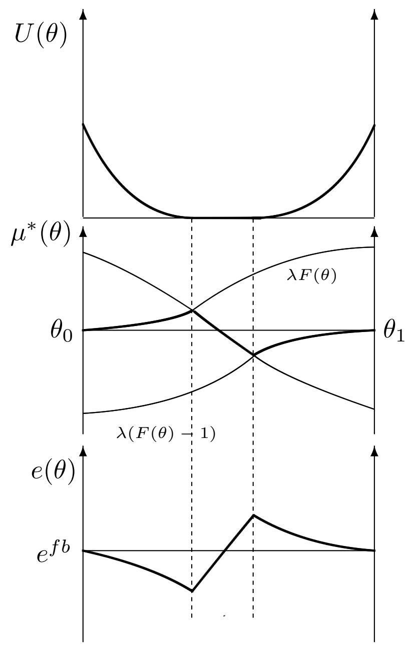

Figure 6: Utility, Costate & Effort Profile – Concave $\pi(\theta, s)$

costate solution induces a switch in the trajectory of the effort profile on partition Θ_{II}. The effort goes from below first best to above in this partition. However, provided that over the entire interval the slope of the effort profile never exceeds 1: $e^*_\theta \leq 1$, then part (ii) of Lemma 1 still holds - the second order condition for the profile to form part of a solution is satisfied. Moreover, since $C_\theta(\theta) \geq 0$, the solution can still be inverted and a net-transfer rule can be derived. However, the rule is decreasing but not convex. The right hand side of Figure 2 shows the non-convexity that is induced in the net-transfer rule. Hence, the linearity property is violated when the reservation utility is concave in cost type. This is analogous to the result in Lewis and Sappington (1989).

6 Conclusion

The objective of this paper was to extend the linearity property of optimal procurement contracts to the more general environment of type-dependent reservation utilities. This situation is likely to arise in the pharmaceutical or defence industries, where firms may have significant market power in a private market or a valuable outside opportunity which they must be compensated for in the procurement contract. In the pharmaceutical industry for instance, this means that lower cost types have higher reservation utility since they forgo greater profits in the private prescription market. Moreover, the higher the quality of the drug under procurement, the greater those foregone profits are likely to be.

The results of this paper showed that in such industries, a countervailing incentive effect must be accommodated in the optimal procurement contract. Furthermore, it was shown that such optimal contracts can only be implemented with a menu of linear contracts if the reservation utility satisfies a certain convexity requirement, in addition to the standard regularity assumptions (as in Laffont and Tirole (1986)). The convexity requirement is robust to various strengths of the countervailing incentive effect. The strength was controlled through the degree of quality, or market power or some other private market parameter, and lead to various sub-cases which induced different rent extraction-efficiency trade-offs.

Concave reservation utility alone does not rule out an optimal menu of contracts, but it does rule out the linearity property. The reason is that concave reservation utility results in the principal switching the trajectory of the effort profile on a subset of cost types, which induces a non-convexity in the net-transfer rule, which if used would result in pooling of cost types. This finding parallels Lewis and Sappington's (1989) result that pooling characterises the equilibrium solution when the reservation utility is concave. However, here a separating contract still exists for concave reservation utility provided that the effort profile is not steeper than unit slope.

The implication for procurement in industries like pharmaceuticals and defence that may exhibit type-dependent reservation utilities is that the linearity property cannot be assumed from the outset without due reference to the curvature of the reservation utility. If the return from under-reporting cost is weakly greater for lower cost firms in those industries, then the results of this paper suggest that the linearity property holds; otherwise optimal contracting does not entail the linearity property.

7 Appendix

7.1 Proof of Lemma 1

The proof is almost identical to Laffont and Tirole (1993, Chapter 2). From the Revelation Principle, attention is restricted to the class of direct truthful mechanisms without loss of generality: $\{y(\theta'), C(\theta')\}_{\theta' \in [\theta_0, \theta_1]}$, where θ' is the agent's report of their cost parameter. Then the agent's payoff as a function of its report is:

$$U(\theta, \theta') = y(\theta') - \psi(\theta - C(\theta')) - \pi(\theta, s)$$

For the necessary and sufficient conditions for local incentive compatibility, every agent θ chooses to report θ' to maximise its transfer from the principal. The local first-order necessary condition is:

$$y_\theta(\theta') - \psi'(\theta - C(\theta'))C_\theta(\theta') = 0 \tag{7.1}$$

Hence, for truth-telling to be optimal for every type, the necessary condition follows by evaluating the previous condition at $\theta = \theta'$:

$$y_\theta(\theta) - \psi'(e(\theta))C_\theta(\theta) \equiv 0 \quad \forall \theta \in \Theta \tag{7.2}$$

As this holds for every type, the condition is an identity. Now differentiating equation (7.1) again with respect to the agent's true type, and evaluating under the condition that the agent reports the truth yields the expression in part (i) of the Lemma:

$$U_\theta(\theta) = -\psi'(e(\theta)) - \pi_\theta(\theta, s)$$

The local second-order condition for truth-telling is:

$$y_{\theta\theta}(\theta) - \psi'(\theta - C(\theta))C_\theta^2 \leq 0$$

Differentiating the local first-order necessary condition for truth-telling (which is identically zero for all θ), the second-order condition is equivalent to:

$$C_\theta(\theta) \geq 0$$

That is, the cost function must be non-decreasing in type.

The requirement for global incentive compatibility follows from the weak-reversal property: for any pair $\theta, \theta' \in \Theta$:

$$U(\theta,\theta) \geq U(\theta,\theta') \quad \& \quad U(\theta',\theta) \geq U(\theta',\theta')$$

Adding these two and integrating gives:

$$\int_{\theta}^{\theta'} \int_{C(\theta)}^{C(\theta')} \psi''(x-y)dxdy \geq 0$$

Since $\psi'' \geq 0$, it follows that provided $\theta \geq \theta'$, then $C(\theta') \geq C(\theta)$. Hence, non-decreasing $C(\theta)$, or $e_\theta \leq 1$ is a necessary condition for incentive compatibility. ■

7.2 Proof of Lemma 2

For partitions Θ_I and Θ_{III}, this is straightforward. Consider partition Θ_{II}. Recall that:

$$\nu_\theta(\theta) = \left(-e^\nu_\theta - \frac{\pi_{\theta\theta}}{\psi''}\right)/e^\nu_\mu \tag{7.3}$$

It is convenient to make the following definition: $V := (1+\lambda)(1-\psi'(e))$, then equation (E) is employed to recover e^ν_μ: First, rearranging gives:

$$\mu(\theta) = Vf/\psi''(e) \tag{7.4}$$

Differentiating this expression with respect to θ yields:

$$\psi''(e^\nu_\theta + e^\nu_\mu \nu_\theta) = \frac{1}{1+\lambda}\left(\frac{\mu f' - \mu_\theta f}{f^2}\right)\psi'' + O(\psi''')$$

Taking $\psi''' \approx 0$ and rearranging this expression to solve for e^ν_μ yields:

$$e^\nu_\mu = \frac{\mu f' - \mu_\theta f - e^\nu_\theta(1+\lambda)f^2}{(1+\lambda)f^2\mu_\theta} \tag{7.5}$$

Where the last equality follows from the definition of V. Now replacing e^ν_μ in equation (7.3) yields:

$$\nu_\theta = \left(f'V + f(1+\lambda)\pi_{\theta\theta}\right)/\psi'' \tag{7.6}$$

To establish that $\nu_\theta \leq \lambda f$, first note that:

$$\begin{aligned}
\nu_\theta - \lambda f &= \frac{Vf' - \pi_{\theta\theta}(1+\lambda)f}{\psi''} - \lambda f \\
&= \frac{f'\mu}{f} + \frac{f(1+\lambda)\pi_{\theta\theta}}{\psi''} - \lambda f \quad \text{(using definition of } V\text{)} \\
&= -f\left(\lambda - \frac{f'\mu}{f^2}\right) + \frac{f(1+\lambda)\pi_{\theta\theta}}{\psi''}
\end{aligned}$$

Now recall $\mu(\theta) \in \{\lambda(F(\theta)-1), \lambda F(\theta)\}$ and that $\frac{d}{d\theta}\left(\frac{\mu}{f}\right) = \lambda - \frac{\mu}{f^2}f' > 0$ by the monotone hazard rate assumption on the distribution of types. So the last line can be written as:

$$\nu_\theta - \lambda f = -f\frac{d}{d\theta}\left(\frac{\mu}{f}\right) + \frac{f(1+\lambda)\pi_{\theta\theta}}{\psi''} \tag{7.7}$$

Now it can be seen from equation (7.7) that if $\pi_{\theta\theta} \leq \psi''\frac{\lambda}{1+\lambda}\frac{d}{d\theta}\left(\frac{F}{f}\right)$, then $\nu_\theta - \lambda f \leq 0$.

To establish that $e_\theta^*(\theta) \leq 0$, recall that the effort profile on partition Θ_{III} depends on the value of the $\nu(\theta, s)$ costate solution, therefore: $\frac{de^\nu}{d\theta} = \frac{\partial e^\nu}{\partial\theta} + \frac{\partial e^\nu}{\partial\nu}\nu_\theta$. Using (7.3), this yields:

$$\frac{de^\nu}{d\theta} = -\frac{\pi_{\theta\theta}}{\psi''} \tag{7.8}$$

So for the effort profile to be downward sloping on partition Θ_{III} requires $\pi_{\theta\theta} \geq 0$ since $\psi'' > 0$ by assumption. ■

7.3 Proof of Lemma 3

Differentiating equation (S) with respect to s yields: $\nu_s = -\frac{\pi_{\theta s}}{\psi'' e_\nu}$. Differentiating equation (E) with respect to ν yields: $\psi'' e_\nu = -\frac{1}{(1+\lambda)f}\psi'' + O(\psi''')$. Hence $e_\nu < 0$. Since $\psi'' > 0$ and $\pi_{\theta s} < 0$ by assumption, then $\nu_s < 0$. That is, the $\nu(\theta, s)$ solution shifts down for greater values of the private market parameter, s. ■

7.4 Proof of Corollary 1

The result follows directly from consideration of the conjectured solution of the costate variable (equation (3.11)), and Figure 4. Since $\nu \in \{\lambda F, \lambda(F-1)\}$, then take (say) $\tilde{\theta} \in int(\Theta_{II})$ in case II. Then it follows that $0 < \nu(\tilde{\theta}) < \lambda F(\tilde{\theta})$. Hence, the distortion implicit in the first order condition for optimal effort is smaller, so e^ν solutions are closer to the first best level. The result is analogous for case IV. ■

7.5 Proof of Corollary 2

First, consider a value of s like s^* such that on an interval of cost types, $U_\theta(\theta) \leq 0$,

$$U(\theta) = \begin{cases} \int_\theta^{\hat{\theta}(s^*)}(\psi' + \pi_\theta)d\tilde{\theta} & \text{for } \theta \leq \hat{\theta}(s^*) \\ 0 & \text{for } \theta \geq \hat{\theta}(s^*) \end{cases}$$

So differentiating this expression with respect to the private market parameter yields:

$$\frac{\partial U(\theta)}{\partial s} = \underbrace{\left[\psi'(e(\hat{\theta}(s^*))) + \pi_\theta(\hat{\theta}(s^*), s^*)\right]}_{(a)} \underbrace{\frac{\partial \hat{\theta}(s^*)}{\partial s}}_{(b)} + \underbrace{\int_\theta^{\hat{\theta}(s^*)} \pi_{\theta s}(\theta, s^*) d\tilde{\theta}}_{(c)} \quad \text{in } \theta \leq \hat{\theta}(s^*)$$

The expression labeled (a) is positive by assumption that $U_\theta(\theta) \leq 0$ - the direct effort incentive dominates the countervailing incentive. To sign the term (b), recall that the critical value $\hat{\theta}$ is the cost type at the border of partition I and II (in this case): $\nu(\hat{\theta}, s^*) = \lambda F(\hat{\theta})$. Differentiating this equation gives the sensitivity of the critical cost type to the private market parameter:

$$\frac{d\hat{\theta}(s)}{ds} = \frac{-\nu_s}{\nu_\theta - \lambda f} < 0$$

since Lemma 2 says that $\nu_\theta \leq \lambda f$ and Lemma 3 says that $\nu_s < 0$. Hence (b) is negative.

Since $\pi_{\theta s}(\theta, s) \leq 0$ by assumption, the integral over this function is negative. Hence (c) is negative. Consequently, an increase in the private market parameter decreases information rents : $U_s(\theta) \leq 0$ when $\pi_{\theta\theta} \in [0, \hat{\pi}_{\theta\theta}]$. The proof for the critical value on the border of partitions II is straightforward, For III, term (a) is negative, (b) is negative, and the endpoints of the integral in (c) are switched, so that (c) is positive. ■

7.6 Proof of Lemma 4

Differentiating equation (5.1) with respect to θ yields:

$$\frac{dY^*}{dc} = y^*_\theta(\theta)\frac{1}{c_\theta} = -\psi'(e^k) \leq 0 \quad \text{and} \quad \frac{d^2Y^*}{dc^2} = -\frac{\psi''(e^k)e^k_\theta}{c_\theta} \geq 0, \quad k = \{L, H, \nu\}$$

where the first order condition for truth-telling outlined in equation (7.2) has been used in the left hand equation, and the result of Lemma 2 has been used in the right hand side equation. Hence, as stated, the net-transfer function is decreasing and convex for each of the separate solution profiles. To show continuity for a patched solution, take type $\hat{\theta}$ on the border of region I and II. Since $\lim_{\theta \to \hat{\theta}^-} e^L(\hat{\theta}) = e^\nu(\hat{\theta})$ and $\lim_{\theta \to \hat{\theta}^+} e^\nu(\theta) = e^L(\hat{\theta})$ then:

$$\lim_{\theta \to \hat{\theta}^-} y^L(\hat{\theta}) = \lim_{\theta \to \hat{\theta}^-} (\psi(e^L(\hat{\theta})) + \pi(\hat{\theta})) = \psi(e^\nu(\hat{\theta})) + \pi(\hat{\theta}) = \lim_{\theta \to \hat{\theta}^+} y^\nu(\hat{\theta})$$

and $\psi(e^L(\hat{\theta})) = \psi(e^\nu(\hat{\theta}))$ exist. Now for differentiability, note that:

$$\lim_{\theta \to \hat{\theta}^-} \frac{dY^*}{dc} = \lim_{\theta \to \hat{\theta}^-} -\psi'(e^L(\hat{\theta})) = -\psi'(e^\nu(\hat{\theta})) = \lim_{\theta \to \hat{\theta}^+} \frac{dY^*}{dc}$$

Also, $\psi'(e^L(\hat{\theta})) = \psi'(e^\nu(\hat{\theta}))$ exist. The same method can be applied to the border of region II and III. ■

7.7 Proof of Lemma 5

Differentiating equation (E) yields:

$$e_\theta^k = -\frac{1}{1+\lambda}\frac{d}{d\theta}\left(\frac{\mu}{f}\right), \qquad k = \{L, H, \nu\}$$

Take $j = L, H$. At $\theta = \hat{\theta}$, from equation (E): $e^j(\hat{\theta}) = e^\nu(\hat{\theta})$ since $\mu^*(\hat{\theta}) = \nu(\hat{\theta})$, but $e_\theta^j \neq e_\theta^\nu$. To see this, note that:

$$\frac{d}{d\theta}\left(\frac{\mu}{f}\right) = \lambda - \frac{\mu}{f^2}f' \quad \text{whereas} \quad \frac{d}{d\theta}\left(\frac{\nu}{f}\right) = \frac{\nu_\theta}{f} - \frac{\nu}{f^2}f'$$

For an interval of types to be binding at the optimum, the required condition is $\nu_\theta \leq \lambda f$. Hence,

$$\frac{d}{d\theta}\left(\frac{\nu(\hat{\theta})}{f}\right) = \frac{\nu_\theta}{f} - \frac{\nu(\hat{\theta})}{f^2}f' \leq \lambda - \frac{\nu(\hat{\theta})}{f^2}f' = \lambda - \frac{\mu(\hat{\theta})}{f^2}f' \Leftrightarrow \frac{d}{d\theta}\left(\frac{\mu(\hat{\theta})}{f}\right) \geq \frac{d}{d\theta}\left(\frac{\nu(\hat{\theta})}{f}\right)$$

Where the first inequality comes from Lemma 2 which says that $\nu_\theta(\theta, s) \leq \lambda f(\theta)$, and the second equality follows from the definition of the costate variable at the border of the partitions: either $\nu(\hat{\theta}) = \lambda F(\hat{\theta})$ or $\nu(\hat{\theta}) = \lambda(F(\hat{\theta}) - 1)$. It follows that $e_\theta^L(\hat{\theta}) \leq e_\theta^\nu(\hat{\theta})$. Now recall that:

$$\frac{d^2Y^*}{dc^2} = -\frac{\psi''(e^k)e_\theta^k}{c_\theta} = -\psi''(e^k)\frac{e_\theta^k}{1-e_\theta^k} = \psi''(e^k)\left|\frac{e_\theta^k}{1-e_\theta^k}\right|, \quad k = \{L, H, \nu\}$$

Since $e_\theta^j < e_\theta^\nu < 0$ for $j = L, H$, then it follows that $\frac{e_\theta^j}{1-e_\theta^j} < \frac{e_\theta^\nu}{1-e_\theta^\nu}$. Again, since the slope of the effort profiles are negative regardless of mode,

$$\left|\frac{e_\theta^j}{1-e_\theta^j}\right| > \left|\frac{e_\theta^\nu}{1-e_\theta^\nu}\right|$$

Using this inequality and the expression for the second cost derivative of the net-transfer function from above, it is straightforward to see that:

$$\frac{d^2Y^j}{dc^2} > \frac{d^2Y^\nu}{dc^2}, \qquad j = L, H$$

Hence, the net-transfer function is less convex on subsets of cost types where the costate solution is $\nu(\theta, s)$ - which is where the IR binds. ■

7.8 Proof of Corollary 3

From equation (4.6) and (7.8),

$$\frac{de^{\nu}}{d\theta} = -\frac{1}{1+\lambda}\frac{d}{d\theta}\left(\frac{\nu}{f}\right) = -\frac{\pi_{\theta\theta}}{\psi''}$$

The result follows from requiring $0 \leq \pi_{\theta\theta} \leq \frac{\psi''}{1+\lambda}\frac{d}{d\theta}(\frac{F}{f})$. ■

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2008-09	Substitution Between Public and Private Consumption in Australian States, **Anna Brown** and **Graeme Wells**

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