Multi-product firms and increasing marginal costs

Oscar Pavlov
University of Tasmania, Australia

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University of Tasmania

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Abstract

Recent literature has addressed how product creation amplifies economic fluctuations via the love of variety. Yet, the empirical evidence on variety effects is sparse. The current paper demonstrates that a decreasing returns to scale production technology, which leads to increasing marginal costs, can similarly amplify business cycles. An expansion of the firm’s product scope reduces marginal costs and gives an incentive to produce multiple products even if the variety effects are entirely absent. The efficiency gains from adjusting product scopes makes the economy more susceptible to sunspot equilibria.

Keywords: Indeterminacy, sunspot equilibria, multi-product firms, business cycles.

JEL Classification: E32.

*Tasmanian School of Business and Economics, University of Tasmania, Sandy Bay TAS 7005, Australia. E-mail address: oscar.pavlov@utas.edu.au.
1 Introduction

An important line of research has demonstrated how product creation via the entry of firms can amplify shocks and be a source of sunspot equilibria that leads to fluctuations driven by self-fulfilling beliefs. The two central mechanisms that produce these results are countercyclical markups and the love of variety.\(^1\) More recently, Minniti and Turino (2013) and Pavlov and Weder (2017) extend these entry models by utilizing variety effects as an incentive for firms to produce multiple products. Yet, the empirical evidence on the size of these effects is sparse - casting doubt on the ability of this mechanism to explain the contribution of product creation on the business cycle. The current paper addresses this issue by laying out a model where intra-firm product creation amplifies business cycles and makes the economy more susceptible to sunspot equilibria even in the complete absence of the love of variety.

Specifically, it investigates the role of increasing marginal costs in a general equilibrium model with endogenous entry and oligopolistic multi-product firms. When the production technology of intermediate good firms has decreasing returns, marginal costs increase with output per variety. This gives firms an incentive to produce multiple products even in the absence of variety effects. The efficiency gains of adjusting product scopes amplifies economic fluctuations and creates sunspot equilibria at more realistic situations, which are not attainable with mono-product firms. Hence, technological decreasing returns and increasing marginal costs provide a novel mechanism for product creation within firms and for generating indeterminacy. This is in stark contrast to Benhabib and Farmer (1994) and Farmer and Guo (1994), where indeterminacy is a result of technological increasing returns for mono-product firms in the absence of entry. Finally, the model is simulated by belief (sunspot) shocks and artificial business cycles closely resemble empirically observed fluctuations.

The way indeterminacy arises is most easily understood in terms of the equilibrium wage-hours locus. Product creation and countercyclical markups generate an endogenous efficiency wedge which makes this locus upwardly sloping. If the locus is steeper than the labor supply curve, then sunspots can act as self-fulfilling expectation shocks. For example, if people become optimistic about the future path of income, then the wealth effect shifts the labor

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\(^1\) For example, under the love of variety, Devereux et al. (1996) assess the effect of technology shocks, while Pavlov and Weder (2012) examine the conditions for local indeterminacy. Jaimovich (2007) investigates how indeterminacy can be generated by oligopolistic firms with countercyclical markups.
supply curve upwards, raising employment and output - thereby confirming the initial belief. More precisely, when the labor supply curve shifts up due to optimistic expectations, the higher demand for output and profit opportunities induce firm entry. Greater competition pushes markups downwards and causes firms to expand output. Since marginal costs would increase with production, firms choose to expand their product scopes rather than ramp up the production of existing varieties. The fall in output per variety due to the cannibalization effect (new varieties reducing the demand for existing varieties) then leads to falling marginal costs. Together, the efficiency gains of product creation and falling markups shift out the labor demand curve far enough to allow the initial belief about higher income to become self-fulfilling.

The paper is most closely related to Pavlov and Weder (2017) who show that under the love of variety, product scope adjustments within firms make the economy more susceptible to sunspot equilibria. In a parallel framework, Minniti and Turino (2013) investigate the magnification effect on fundamental disturbances only. In contrast, the current paper does not utilize variety effects. Instead, increasing marginal costs are an incentive for firms to expand their product scopes.

The focus on multi-product firms is motivated by recent empirical work. Bernard et al. (2010) find that about 90 percent of total sales in the manufacturing sector are made by multi-product firms. Broda and Weinstein (2010) report that over 90 percent of product creation and destruction occurs within firms and that the contribution of product scope adjustments within firms is at least as important to the evolution of aggregate output as firm entry and exit.

The paper proceeds as follows. Section 2 outlines the model. Section 3 analyzes the local dynamics. Capital utilization is introduced in Section 4 and the model is simulated in Section 5. Section 6 concludes.

2 Model

The artificial economy is based on the multi-product models of Minniti and Turino (2013) and Pavlov and Weder (2017). There are two key differences. First, the love of variety effects are entirely absent in aggregating intermediate goods into final goods. Second, a decreasing
returns to scale production technology implies that marginal costs increase with production. Firms therefore take into account the effect of their product scope decision on marginal costs.

### 2.1 Final goods

Final output, $Y_t$, is produced under perfect competition using the range of intermediate inputs supplied by $M_t$ multi-product firms. This is done via two nested CES aggregators. The first combines the varieties from an individual firm

$$Y_t(i) = N_t(i)^{\frac{1}{1-\sigma_0}} \left( \int_0^{N_t(i)} y_t(i,j)^{\frac{\theta}{\sigma_0}} dj \right)^{\frac{\theta}{\sigma_0-1}} \quad \theta > 1$$

(1)

where $N_t(i)$ is firm $i$’s product scope, $y_t(i,j)$ is the amount of the unique intermediate good $j$ produced by firm $i$, and $\theta$ is the elasticity of substitution. The firm-composite goods are then aggregated to form the final good

$$Y_t = M_t^{\frac{1}{1-\sigma_0}} \left( \sum_{i=1}^{M_t} Y_t(i)^{\frac{\theta}{\sigma_0}} \right)^{\frac{\theta}{\sigma_0-1}}.$$  

(2)

Note that the CES aggregators have been formed to eliminate the love of variety.\footnote{As we will see later, an intermediate good firm will charge the same price for all of its varieties and produce them in equal quantities. Together with the elimination of the love of variety, this implies that differences in intra-firm and inter-firm elasticities of substitution are irrelevant for dynamics.} Hence, unlike in Minniti and Turino (2013) and Pavlov and Weder (2017) where variety effects were necessary for the existence of multi-product firms, the current model has no increasing returns in the aggregation of products.

The profit maximization problem yields

$$y_t(i,j) = \left( \frac{p_t(i,j)}{P_t} \right)^{-\theta} \frac{Y_t}{M_t N_t(i)}$$

(3)

where the aggregate price index satisfies

$$P_t = M_t^{\frac{1}{1-\sigma_0}} \left( \sum_{i=1}^{M_t} P_t(i)^{1-\theta} \right)^{\frac{1}{1-\sigma_0}}.$$  

(4)

In addition, we have the price index for firm $i$’s goods

$$P_t(i) = N_t(i)^{\frac{1}{1-\sigma_0}} \left( \int_0^{N_t(i)} p_t(i,j)^{1-\theta} dj \right)^{\frac{1}{1-\sigma_0}}.$$  

(5)
2.2 Intermediate good firms

Each firm chooses the number of different products to produce and the prices to sell them at. This task is solved in two stages. In the first, firms decide their product scopes. In the second stage, firms act as Bertrand competitors in the product market and set their prices. The model is solved by backward induction using the subgame Nash perfect equilibrium concept. The number of active firms is determined by a zero-profit condition each period. Since all firms have the same technology and behavior is governed by identical optimality conditions, a symmetric Nash equilibrium emerges.

Intermediate goods are produced using capital, \( k_t(i,j) \), and labor, \( h_t(i,j) \), that are supplied on perfectly competitive factor markets. The production technology has decreasing returns and involves two fixed costs. The variety-level fixed cost, \( \phi \), restricts the amount of varieties a firm will produce. The firm-level fixed cost, \( \phi_f \), provides economies of scope and determines the number of active firms via a zero-profit condition. Hence, a firm’s output is given by

\[
\int_0^{N_t(i)} \gamma_t(i,j) \, dj = \int_0^{N_t(i)} [(k_t(i,j)^\alpha h_t(i,j)^{1-\alpha})^\eta - \phi] \, dj - \phi_f \quad 0 < \alpha < 1.
\]

The presence of decreasing returns via \( 0 < \eta < 1 \) makes it profitable for firms to produce multiple products. Each firm sets prices to maximizes profits

\[
\pi_t(i) = \int_0^{N_t(i)} p_t(i,j)\gamma_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j) \, dj
\]

where \( w_t \) and \( r_t \) are the labor and capital rental rates. As in Yang and Heijdra (1993), intermediate good firms are large enough to take the aggregate price index into consideration when making their pricing decision.\(^3\) Appendix A.1 shows that a firm charges the same price, \( p_t(i) \), for all of its varieties and the markup becomes

\[
\mu_t(i) = \frac{\theta[1 - \epsilon_t(i)]}{\theta[1 - \epsilon_t(i)] - 1}
\]

where \( mc_t(i) \) is the marginal cost and \( \epsilon_t(i) \equiv P_t(i)Y_t(i)/(P_tY_t) \) is firm \( i \)'s market share which increases in the number of goods \( N_t(i) \). In contrast to Minniti and Turino (2013) and others, this does not arise from the love of variety but instead is due to the effect of the product

\(^3\)Under monopolistic competition where firms take the aggregate price index as given, the markup and product scope are constant over the business cycle (see Appendix A.3).
scope on the firm’s marginal costs. If the production technology had constant returns to scale (minus the fixed costs), the marginal cost of producing an additional unit of a variety would be independent of the scale of production. Firms would take the marginal cost as given when making their product scope decisions. Profits would be decreasing in \( N_t(i) \) because of the variety-level fixed cost, \( \phi \), and firms would only produce a single product. However, since the production technology has decreasing returns, the marginal cost is increasing in the scale of production:

\[
mc_t(i) = \left( y_t(i) + \phi + \frac{\phi_f}{N_t(i)} \right)^{\frac{1}{\eta}} \frac{w_t^{1-\alpha} r_t^{\alpha}}{\eta(1 - \alpha)^{1-\alpha} \alpha^\alpha}.
\]

Firms then need to take into account how an introduction of a new product affects marginal costs through the demand for their varieties.

Firms determine their optimal number of products by maximizing profits with respect to \( N_t(i) \) by taking into account the effect on its own and other firms’ pricing decisions (see Appendix A.2). The first-order condition is

\[
m_{c_t}(i) \eta \phi = \left( 1 - \eta + \eta \phi \left( \frac{p_t(i) - mc_t(i)}{p_t(i)} \right)^2 \right) P_t Y_t \frac{\partial \epsilon_t(i)}{\partial N_t(i)} - \eta [N_t(i) \phi + \phi_f] \frac{\partial mc_t(i)}{\partial N_t(i)} + Y_t \epsilon_t(i) \left( \frac{p_t(i) - mc_t(i) \eta}{p_t(i)} \right) \frac{\partial P_t}{\partial N_t(i)}.
\]

Firms equate the cost of producing a new variety (the left-hand side) with the gains on the right-hand side. The first two terms on the right-hand side are due to the presence of decreasing returns: introducing a new product increases the firm’s market share, \( \partial \epsilon_t(i)/\partial N_t(i) > 0 \), due to the effect on marginal costs, \( \partial mc_t(i)/\partial N_t(i) < 0 \). The last term represents that introducing a new product reduces the aggregate price index, \( \partial P_t/\partial N_t(i) < 0 \), which from (3) leads to a lower demand for firm \( i \)’s products.

### 2.3 Symmetric equilibrium

In the symmetric equilibrium, each firm produces the same number of varieties, \( N_t(i) = N_t \), charges the same price, \( p_t(i) = p_t \), and has the same market share \( \epsilon_t(i) = 1/M_t \). With the final good as the numeraire, \( P_t = 1 \), and from (4) and (5), \( p_t = P_t \). Using (1) and (2), output per variety is

\[
y_t = \frac{Y_t}{N_t M_t}.
\]
The markup simplifies to
\[ \mu_t = \frac{\theta(M_t - 1)}{\theta(M_t - 1) - M_t}. \]  
(10)

Since new entrants reduce firms’ market shares, the markup is countercyclical. Note that as the number of firms becomes large, the steady state markup converges to its monopolistic competition level of \( \mu = \frac{\theta}{(\theta - 1)} \). Furthermore, the steady state version of this equation can be written as
\[ M = 1 + \frac{\mu}{\mu(\theta - 1) - \theta} \]
and calibrating the steady state markup \( \mu \) and elasticity \( \theta \) pins down the number of firms.

An increase in the firm’s product scope reduces its own price and the prices of other firms: to lower price competition, firms under-expand their product scopes in comparison to the case of monopolistic competition where such strategic linkages are absent. The extent of this under-expansion can be seen by substituting \( \partial e_t(i) / \partial N_t(i) \), \( \partial mc_t(i) / \partial N_t(i) \) and \( \partial P_t / \partial N_t(i) \) into (8) and rearranging for the product scope:
\[ N_t = \frac{1 - \eta \mu_t Y_t}{\eta \phi M_t \Theta_t}. \]

The function \( \Theta_t \) (see Appendix A.2) is less than one and is increasing in \( M_t \): the strategic effect of the product scope decision becomes less important as the number of firms increases and this gives an incentive to introduce new varieties. When \( M_t \) becomes very large this term approaches unity and the markup converges to its monopolistic competition level of \( \theta / (\theta - 1) \). Intuitively, as the number of firms grows, the impact on the market share of adding an additional variety becomes smaller, which has then a smaller impact on the price of the variety. The dynamics of the product scope are thus similar to Pavlov and Weder (2017), but instead of the love of variety being the incentive for product creation, it is the efficiency gains of reducing marginal costs. When firms want to expand their output it is efficient to introduce new products, rather than ramp up the production of existing varieties whose production technology is subject to diminishing returns.

The number of firms can be determined from the zero profit condition:
\[ M_t = \frac{\mu_t - \eta}{\eta \phi N_t + \phi_f} \]  
(11)
To obtain aggregate output, first note that (6) can be written as
\[ y_t = k_t^\alpha k_t^{\eta(1-\alpha)} - \phi \frac{\phi_f}{N_t}. \]
Using this together with (9) and (11) gives
\[ Y_t = \eta \frac{M_t^{1-\eta} N_t^{1-\eta}}{\mu_t} K_t^{\eta \alpha} H_t^{\eta (1-\alpha)} \]  
(12)

where \( K_t = M_t N_t k_t \) and \( H_t = M_t N_t k_t \). The term \( \frac{M_t^{1-\eta} N_t^{1-\eta}}{\mu_t} \) can be interpreted as an endogenous efficiency wedge. Combining (11) and (12), output can be written as
\[ Y_t = \frac{\eta}{\mu_t} K_t^{\alpha} H_t^{1-\alpha} \left( 1 - \frac{\eta}{\mu_t} \right) \left( \frac{N_t}{\phi N_t + \phi_f} \right)^{\frac{1-\eta}{\eta}}. \]
(13)

Since markups are countercyclical, the first term in brackets implies that decreasing returns \((\eta < 1)\) have a contractionary effect on the efficiency wedge. However, the second term in brackets implies that an increase in product scopes has an expansionary effect. This latter effect outweighs the former and as we will see in Section 3, procyclical product scope makes the economy more susceptible to sunspot equilibria. Finally, the equilibrium real wage and rental rate are given by
\[ w_t = (1 - \alpha) \frac{Y_t}{H_t} \quad \text{and} \quad r_t = \alpha \frac{Y_t}{K_t}. \]

### 2.4 Agents

The representative agent derives lifetime utility from the function
\[ U = \int_0^\infty e^{-\rho t} u(C_t, H_t)dt \quad \rho > 0. \]

Here, \( \rho \) denotes the subjective rate of time preference and period utility takes the functional form
\[ u(C_t, H_t) = \ln C_t - v \frac{H_t^{1+\chi}}{1+\chi} \quad v > 0, \ \chi \geq 0 \]

where \( \chi \) is the inverse of the Frisch labor supply elasticity. The agents own the capital stock and sell labor and capital services. The period budget is constrained by
\[ w_t H_t + r_t K_t + \Pi_t \geq X_t + C_t \]

where \( \Pi_t \) denotes potential profits and investment, \( X_t \), is added to the capital stock such that:
\[ \dot{K}_t = X_t - \delta K_t \quad 0 < \delta < 1. \]
Time derivatives are denoted by dots and $\delta$ stands for the constant rate of physical depreciation of the capital stock. The solution to the maximization problem gives

$$\nu H_t^x = \frac{w_t}{C_t}$$  \hspace{1cm} (14)$$

and

$$\frac{\dot{C}_t}{C_t} = r_t - \delta - \rho.$$  \hspace{1cm} (15)$$

Equation (14) describes the agents’ leisure-consumption trade-off, while (15) is the intertemporal Euler equation. In addition the transversality condition must hold.

### 3 Dynamics

This section analyzes the local dynamic properties of the multi-product model and compares it to the mono-product model. The equilibrium conditions are log-linearized and the dynamical system is arranged to

$$\begin{bmatrix} \dot{K}_t/K_t \\ \dot{C}_t/C_t \end{bmatrix} = J \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}.$$  

Hatted variables denote percent deviations from their steady-state values and $J$ is the $2 \times 2$ Jacobian matrix of partial derivatives. Note that $C_t$ is a non-predetermined variable and that $K_t$ is predetermined. Indeterminacy requires both roots of $J$ to be negative, that is $\text{Det}\ J > 0 > \text{Tr}\ J$. For easier comparison to previous studies, the parameters are calibrated at a quarterly frequency as $\alpha = 0.3$, $\rho = 0.01$, $\delta = 0.025$ and $\chi = 0$.

As explained in the previous section, the current model requires decreasing returns, $0 < \eta < 1$, for firms to have an incentive to produce multiple products. Figure 1 presents the indeterminacy zones for the mono-product model with $\eta = 1$ and the multi-product model with $\eta = 0.8$ and $0.9$, which is consistent with the evidence in Burnside (1996). The mono-product model is virtually identical to Jaimovich (2007) if, in his paper, the intersectoral elasticity of substitution is set to unity. Indeterminacy is driven entirely by the effect of the countercyclical markup on the efficiency wedge. While not shown in the figure, decreasing returns in the mono-product model reduce the plausibility of indeterminacy. The reason is that falling markups increase the output of firms and then the diminishing returns have a negative effect on the efficiency wedge (recall the first bracketed term in equation 13). As $\theta$
Figure 1: Mono and multi-product models. Shaded regions represent areas of indeterminacy.

falls towards $\mu/(\mu - 1)$ the number of firms approaches infinity and the markup converges to its monopolistic competition level of $\theta/(\theta - 1)$. In this case, the markup is constant, the dynamics converge to that of the model with monopolistic competition, and indeterminacy cannot exist. Higher $\theta$, on the other hand, increases the cyclicality of the markup and indeterminacy is thus possible for a lower level of market power.

When decreasing returns are present, marginal costs increase with output per variety and firms have an incentive to produce multiple products. Here, decreasing returns amplify fluctuations and reduce the level of market power required for indeterminacy. This stands completely in contrast to how decreasing returns affect the mono-product model. Why is this the case? First, note that if markups were constant, the product scope, output per firm and output per variety would also be constant: $\eta$ would have no effect on local dynamics (see Appendix A.3). Under oligopolistic competition, however, the entry of new firms reduces existing firms’ market shares and encourages them to expand their product scopes. Due to the cannibalization effect (new products reducing demand for existing products), firms reduce production for each of their varieties, which then reduces their marginal costs. This efficiency gain - firms do not run into diminishing returns as quickly as their mono-product
counterparts - acts as an additional mechanism that amplifies business cycles. That is, a low number of firms leads to two inefficiencies: high markups and low product scopes (with high output per variety). Firm entry reduces these inefficiencies and expands production possibilities.

4 Capital utilization

The last section has demonstrated that when production technologies have diminishing returns, the possibility of sunspot equilibria increases when firms can choose their product scopes. However, it could be argued that the level of market power required for indeterminacy is on the higher end of empirical estimates. This section addresses the issue by showing that the levels of market power can be reduced substantially by introducing variable capital utilization. Each intermediate good firm $i$ now operates the production technology

$$\int_0^{N(i)} y_t(i, j) dj = \int_0^{N(i)} \left[ (U_t^\alpha k_t(i, j)^\alpha h_t(i, j) (1-\alpha)^\eta - \phi_j \right] dj - \phi_f$$

where $U_t$ stands for the utilization rate of capital set by its owners. Capital evolves according to

$$\dot{K}_t = X_t - \delta_t K_t = X_t - \frac{1}{\rho} U_t^\rho K_t \quad \rho > 1.$$

and the optimal rate of utilization follows

$$r_t = U_t^{\rho-1}.$$

The calibration remains the same, and as in Wen (1998), the steady state first-order conditions pin down $\rho = (\rho + \delta)/\delta = 1.4$. Figure 2 demonstrates how the introduction of variable capital utilization significantly reduces the level of market power and the elasticity of substitution that are required for indeterminacy. This occurs because higher utilization, like lower markups, increases the demand for labor.

To gain further understanding about the effect of sunspots and the dynamics of the model, the impulse responses of the main variables are plotted in Figure 3. The sunspot shock is modelled as an expectation shock to consumption that raises it one percent above its steady state level. This discrete-time version of the model is calibrated as $\alpha = 0.3$, $\delta = 0.025$, $\chi = 0$, $\eta = 0.9$, and a discount factor at $\beta \equiv (1 + \rho)^{-1} = 0.99$. The steady markup is
Figure 2: Multi-product model with variable capital utilization, $\eta = 0.9$.

set to $\mu = 1.3$, which lies in the middle of value-added markup estimates for the US (see Jaimovich, 2007). Finally, following Minniti and Turino (2013) and Pavlov and Weder (2017) the elasticity of substitution is set to $\theta = 7.5$. The impulse response functions reveal that both net product creation and net business formation positively comove with output, with the former being more volatile than the latter. We can also observe the cannibalization effect: an introduction of a new variety reduces the demand for existing varieties, that is, output per variety drops. The countercyclically fluctuating markup, together with the efficiency gains of product creation on marginal costs leads to an upwardly sloping wage-hours locus that enables the propagation of self-fulfilling beliefs described earlier.

5 Simulations

[This section is highly incomplete and subject to major revisions]

So far, this paper has shown that under increasing marginal costs, intra-firm product creation can generate indeterminacy under more plausible situations. This remains the case even if the love of variety - typically necessary for the existence of multi-product firms - is entirely absent. The current section simulates the model to see if it can replicate the
Figure 3: Impulse responses to a consumption (sunspot) shock (percent deviations from the steady state).

basic business cycle facts by comparing its second moments to the US quarterly time series counterparts. Finally, a comparison is made to the model of Pavlov and Weder (2017) where variety effects drive the creation of products.

A discrete time version of the model with capital utilization is simulated by i.i.d. sunspot shocks only. The calibration remains as in the previous section. Table 1 presents HP-filtered second moments of the US data and of the artificial economy. The model correctly reproduces the order of relative volatilities and positive correlations for the main macroeconomic aggregates and the markup is strongly negatively correlated with output.\footnote{The low volatility of consumption is the consequence of the utilization margin and has been noted by Jaimovich (2007) and Wen (1998). It can be shown that a steeper wage-hours locus via a lower $\eta$ or a more cyclical markup (higher $\mu$ and/or $\theta$) improves the model’s performance in this aspect.} The statistics for the firm dynamics should be taken with a grain of salt as the data for net product creation is limited to 2000:I – 2003:IV, while the difference between openings and closings of establishments provides only a crude measure of net business formation. Yet, the model does well in replicating these observations. That is, both net product creation and net business formation are positively correlated with output, with the former being more volatile than
The latter.

The last two columns of Table 1 present the second moments of the multi-product model of Pavlov and Weder (2017). There are two main differences from the current model. First, there are no decreasing returns in production, i.e. $\eta = 1$. Second, the love of variety effects amplify fluctuations. The inter-firm elasticity remains at $\theta = 7.5$ but the intra-firm elasticity is $\gamma = 11.5$ (following Broda and Weinstein, 2010) which imply variety effects of $1/(\theta - 1)$ and $1/(\gamma - 1)$. Apart from this, the calibration remains the same. To get a better understanding of the workings of the model, the functional form of their CES aggregators imply that there are increasing returns in the aggregation of products:

$$Y_t = M_t^{\theta/(\theta-1)} N_t^{\gamma/(\gamma-1)} y_t$$

and their aggregate production function can be written as

$$Y_t = \frac{M_t^{1/(\theta-1)} N_t^{1/(\gamma-1)}}{\mu_t} (U_t K_t)^{\alpha} H_t^{1-\alpha}.$$

As can be seen from the table, the model performs very comparably to the decreasing returns economy. Since the two variety effects produce a steeper wage-hours locus, this version of the model does a slightly better job at matching the volatility of consumption. Overall, it can be concluded that increasing marginal costs work similarly to the love of variety effects in a setting with multi-product firms.

<table>
<thead>
<tr>
<th>Variable $x$</th>
<th>U.S. data $\sigma_x/\sigma_Y$</th>
<th>Model ($\eta = 0.9$) $\rho(x, Y)$</th>
<th>PW (2017) $\rho(x, Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>consumption</td>
<td>0.39</td>
<td>0.82</td>
<td>0.09</td>
</tr>
<tr>
<td>investment</td>
<td>2.63</td>
<td>0.98</td>
<td>4.42</td>
</tr>
<tr>
<td>hours worked</td>
<td>0.83</td>
<td>0.86</td>
<td>0.93</td>
</tr>
<tr>
<td>net business formation</td>
<td>0.23</td>
<td>0.26</td>
<td>0.28</td>
</tr>
<tr>
<td>net product creation</td>
<td>1.08</td>
<td>0.42</td>
<td>0.67</td>
</tr>
<tr>
<td>markup</td>
<td>-</td>
<td>-</td>
<td>0.11</td>
</tr>
</tbody>
</table>

$\sigma_Y$ denotes the standard deviation of output and $\rho(x, Y)$ is the correlation of variable $x$ and output. See Appendix A.4 for the source of US data. Net product creation data, limited to the period 2000:I to 2003:IV, is from Broda and Weinstein (2010). Blank entries for the markup series are due to data unavailability.
6 Conclusion

Previous studies have shown that product creation within firms can be a source of business cycle amplification and sunspot equilibria. Yet, this result and the existence of multi-product firms relies on the love of variety, for which empirical evidence is limited. The current paper addresses this issue. It investigates the role of increasing marginal costs in a dynamic general equilibrium model without the love of variety. Marginal costs increase with output per variety due to decreasing returns in the production technology. Product scope expansions then reduce marginal costs and firms have an incentive to produce multiple products. The efficiency gains of adjusting product scopes provides an amplification mechanism that creates sunspot equilibria at more realistic situations, which are not attainable with mono-product firms. Hence, increasing marginal costs provide a novel mechanism for product creation and make it easier for indeterminacy to occur. The simulated indeterminate model generates artificial business cycles that closely resemble empirically observed fluctuations.

References


A Appendix

A.1 Markups

This Appendix derives the intermediate good firm’s optimal markup. Taking logs of (3) gives

\[
\ln y_t(i, j) = -\theta \ln p_t(i, j) + \theta \ln P_t + \ln Y_t - \ln N_t(i) - \ln M_t.
\]

Then using (4) and (5), the price elasticity of demand is

\[
\frac{\partial \ln y_t(i, k)}{\partial \ln p_t(i, j)} = \underbrace{-\theta}_{\text{absent for } k \neq j} + \frac{\theta}{N_t(i) M_t} \left( \frac{p_t(i, j)}{P_t} \right)^{1-\theta}.
\]  

(A.1)

Firm \( i \) maximizes profit (7) subject to the constraint (6):

\[
\mathcal{L} = \int_0^{N_t(i)} p_t(i, j) y_t(i, j) - w_t h_t(i, j) - r_t k_t(i, j) dj \\
+ \Lambda_t \left( \int_0^{N_t(i)} \left[ z_t(k_t(i, j)^{a} h_t(i, j)^{1-a})^\eta - \phi \right] dj - \phi_f - \int_0^{N_t(i)} y_t(i, j) dj \right).
\]
Optimality gives

$$\frac{\partial L}{\partial p_t(i,j)} = y_t(i,j) + \int_0^{N_t(i)} [p_t(i,j) - \Lambda_t] \frac{\partial y_t(i,j)}{\partial p_t(i,j)} dj = 0 \quad (A.2)$$

$$\frac{\partial L}{\partial h_t(i,j)} = -w_t + \Lambda_t \eta (1 - \alpha) z_t k_t(i,j)^{\alpha \eta} h_t(i,j)^{(1-\alpha)\eta - 1} = 0 \quad (A.3)$$

$$\frac{\partial L}{\partial k_t(i,j)} = -r_t + \Lambda_t \eta \alpha z_t k_t(i,j)^{\alpha \eta - 1} h_t(i,j)^{(1-\alpha)\eta} = 0. \quad (A.4)$$

The Lagrange multiplier, $\Lambda_t$, is obtained by combining (A.3) and (A.4) then applying Shephard’s lemma, and amounts to the marginal cost, $mc_t(i,j)$ of producing one more variety:

$$mc_t(i,j) = (z_t k_t(i,j)^{\alpha \eta} h_t(i,j)^{(1-\alpha)\eta})^{\frac{1-\eta}{\eta}} \frac{w_t^{1-\alpha} r_t^\alpha}{z_t^\eta \eta (1-\alpha)^{1-\alpha} \alpha} \quad (A.5)$$

Substituting (A.1) into (A.2) and some algebra yields

$$y_t(i,j) - \theta \frac{y_t(i,j)}{p_t(i,j)} [p_t(i,j) - mc_t(i,j)] + \int_0^{N_t(i)} y_t(i,k) \frac{y_t(i,k)}{p_t(i,j)} [p_t(i,k) - mc_t(i,k)] dk \frac{\theta}{N_t(i)} \frac{p_t(i,j)}{M_t} (\frac{p_t(i,j)}{P_t})^{1-\theta} = 0$$

Substituting (3) for $y_t(i,j)$, the above equation simplifies to

$$P_t Y_t \left[ 1 - \theta \frac{p_t(i,j)}{p_t(i,j)} - mc_t(i,j) \right] + \theta \int_0^{N_t(i)} y_t(i,k) [p_t(i,k) - mc_t(i,k)] dk = 0$$

As the second term of this equation is the same for all $j \in [0, N_t(i)]$, this implies that firm $i$ will charge the same price for all of its varieties.\textsuperscript{5} Hence, $p_t(i,j) = p_t(i,k) = p_t(i) = P_t(i)$ and $mc_t(i,j) = mc_t(i)$. Some algebra gives

$$\mu_t(i) \equiv \frac{p_t(i)}{mc_t(i)} = \frac{\theta[1 - \epsilon_t(i)]}{\theta[1 - \epsilon_t(i)] - 1} \quad (A.6)$$

where

$$\epsilon_t(i) \equiv \left( \frac{p_t(i)}{P_t} \right)^{1-\theta} M_t^{-1} = \frac{P_t(i) Y_t(i)}{P_t Y_t} \quad (A.7)$$

is firm $i$’s market share.

\textsuperscript{5}Marginal cost depends on the level of production if $\eta \neq 1$ but note that each variety faces the same demand curve.
A.2 Product scope

This Appendix derives the firms’ optimal product scope assuming increasing marginal costs, \( \eta < 1 \). Since the firm will charge the same price for all of its varieties, it will produce the same quantity of each variety. Hence, the costs of production are

\[
\int_0^{N_t(i)} w_t h_t(i, j) + r_t k_t(i, j) \, dj = \eta N_t(i) mc_t(i) z_t k_t(i)^{\alpha} h_t(i)^{(1-\alpha)\eta} = \eta N_t(i) mc_t(i) y_t(i) + N_t(i) \phi + \phi_f .
\]

Profits can then be written as

\[
\pi_t(i) = \left( \frac{p_t(i) - mc_t(i)\eta}{p_t(i)} \right) P_t Y_t \epsilon_t(i) - mc_t(i)\eta [N_t(i) \phi + \phi_f] .
\]

(A.8)

Firm \( i \) takes the number of firms and their product scopes as given and maximizes its profits with respect to \( N_t(i) \) by taking account the effect of its product scope decision on its own and all other producers’ prices and marginal costs.\(^6\)

After some algebra, the first-order condition is

\[
\frac{\partial \pi_t(i)}{\partial N_t(i)} = \left( 1 - \eta + \eta \theta \left( \frac{p_t(i) - mc_t(i)}{p_t(i)} \right)^2 \right) P_t Y_t \frac{\partial \epsilon_t(i)}{\partial N_t(i)} - mc_t(i)\eta \phi
\]

(A.9)

\[
+ Y_t \epsilon_t(i) \left( \frac{p_t(i) - mc_t(i)\eta}{p_t(i)} \right) \frac{\partial P_t}{\partial N_t(i)} - \eta [N_t(i) \phi + \phi_f] \frac{\partial mc_t(i)}{\partial N_t(i)} = 0 .
\]

Now to derive \( \partial \epsilon_t(i)/\partial N_t(i), \partial P_t/\partial N_t(i), \partial mc_t(i)/\partial N_t(i) \), then substitute in (A.9) to obtain firm \( i \)'s product scope. From (A.7):

\[
\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = (1 - \theta) \frac{\epsilon_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} + (\theta - 1) \frac{\epsilon_t(i)}{P_t} \frac{\partial P_t}{\partial N_t(i)} .
\]

(A.10)

Note that the second term on the right hand side of (A.10) would not be present in the case of monopolistic competition. As will be shown later, \( \partial p_t(i)/\partial N_t(i) \) and \( \partial P_t/\partial N_t(i) \) are both negative. From (A.6):

\[
\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{\mu_t(i)[\mu_t(i) - 1]mc_t(i)}{1 - \epsilon_t(i)} \frac{\partial \epsilon_t(i)}{\partial N_t(i)} + \mu_t(i) \frac{\partial mc_t(i)}{\partial N_t(i)} .
\]

(A.11)

Since \( p_t(i) = P_t(i) \), the aggregate price index can be written as

\[
P_t = M_t^{1-\theta} \left( \sum_{k=1}^{M_t} p_t(k)^{1-\theta} \right)^{\frac{1}{1-\theta}}
\]

\(^6\)Note that from (A.5) if \( \eta \neq 1 \) then the firm internalises the effect of the product scope on its marginal costs.
and
\[ \frac{\partial P_t}{\partial N_t(i)} = P_t^\theta M_t^{-1} \left[ \sum_{k=1}^{M_t} \frac{\partial p_t(k)}{\partial N_t(i)} \right]. \tag{A.12} \]

Under symmetry where all firms start off identical with \( p_t(i) = p_t(k) = p_t \), this is equal to
\[ \frac{\partial P_t}{\partial N_t(i)} = \left( \frac{p_t}{P_t} \right)^{-\theta} M_t^{-1} \left[ (M_t - 1) \frac{\partial p_t(k)}{\partial N_t(i)} + \frac{\partial p_t(i)}{\partial N_t(i)} \right] \tag{A.13} \]

and using (A.11) can be written as\(^7\)
\[ \frac{\partial P_t}{\partial N_t(i)} = \left( \frac{p_t}{P_t} \right)^{-\theta} \frac{\mu_t}{M_t} \left[ (M_t - 1) \frac{\partial m_c(k)}{\partial N_t(i)} + \frac{\partial m_c(i)}{\partial N_t(i)} \right] \tag{A.14} \]

From (A.5)\(^8\)
\[ \frac{\partial m_c(i)}{\partial N_t(i)} = \frac{1 - \eta}{\eta} m_c(i) \left( y_t(i) + \phi + \frac{\phi_f}{N_t(i)} \right)^{-1} \times \left[ -\frac{y_t(i)}{N_t(i)} - \frac{\delta y_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} + \theta \frac{y_t(i)}{p_t(i)} \frac{\partial P_t}{\partial N_t(i)} - \frac{\phi_f}{(N_t(i))^2} \right] \]
\[ \frac{\partial m_c(k)}{\partial N_t(i)} = \frac{1 - \eta}{\eta} m_c(k) \left( y_t(k) + \phi + \frac{\phi_f}{N_t(k)} \right)^{-1} \times \left[ -\frac{y_t(k)}{N_t(k)} - \frac{\delta y_t(k)}{p_t(k)} \frac{\partial p_t(k)}{\partial N_t(i)} + \theta \frac{y_t(k)}{p_t(k)} \frac{\partial P_t}{\partial N_t(i)} \right] \]

Now assuming symmetry, setting the price index as the numeraire \( P_t = p_t = 1 \), and using (A.13):
\[ \frac{\partial m_c(i)}{\partial N_t(i)} = \frac{1 - \eta}{\eta} m_c \left( y_t + \phi + \frac{\phi_f}{N_t} \right)^{-1} \times \left[ -\frac{y_t}{N_t} - \frac{\phi_f}{N_t^2} + \theta y_t \frac{M_t - 1 \frac{\partial p_t(k)}{\partial N_t(i)} + 1 - M_t \frac{\partial p_t(i)}{\partial N_t(i)}}{M_t \frac{\partial N_t(i)}{\partial N_t(i)}} \right] \]
\[ \frac{\partial m_c(k)}{\partial N_t(i)} = \frac{1 - \eta}{\eta} m_c \left( y_t + \phi + \frac{\phi_f}{N_t} \right)^{-1} \theta y_t \left[ -\frac{1}{M_t} \frac{\partial p_t(k)}{\partial N_t(i)} + 1 + \frac{\partial p_t(i)}{\partial N_t(i)} \right] \]

Now use these in (A.14) to get
\[ \frac{\partial P_t}{\partial N_t(i)} = \frac{1}{M_t} \frac{\eta - 1}{\eta} \left( \frac{y_t + \phi}{N_t} + \frac{\phi_f}{N_t^2} \right)^{-1} \left( \frac{y_t}{N_t} + \frac{\phi_f}{N_t^2} \right) \]

\(^7\)Note that \( \sum_{k=1}^{M_t} \epsilon_t(k) = 1 \). Then \( \sum_{k=1}^{M_t} \frac{\partial \epsilon_t(k)}{\partial N_t(i)} = 0 \), which under symmetry is \( (M_t - 1) \frac{\partial \epsilon_t(k)}{\partial N_t(i)} + \frac{\partial \epsilon_t(i)}{\partial N_t(i)} = 0 \).

\(^8\)Since the marginal cost and price is the same for all firm \( i \)'s varieties, \( y_t(i, j) = y_t(i, k) = y_t(i) \).
Manipulating the symmetric equilibrium version of the zero profit condition (A.8) gives
\[
\frac{\phi M_t N_t + \phi_f M_t}{Y_t} = \frac{\mu_t}{\eta} - 1.
\]
Then, noting that \( y_t = \frac{Y_t}{M_t N_t} \), the above simplifies to
\[
\frac{\partial P_t}{\partial N_t(i)} = \frac{(\eta - 1)(1 + \Phi_t^f)}{\mu_t M_t N_t(i)} < 0
\]
where \( \Phi_t^f \equiv \frac{\phi f}{Y_t} \) is the share of firm-level fixed costs in final output. Similar to the models with the love of variety, an expansion of the product scope reduces the aggregate price index. \( \frac{\partial mc_t(i)}{\partial N_t(i)} \) can be rearranged to
\[
\frac{\partial mc_t(i)}{\partial N_t(i)} = \frac{\eta - 1}{\mu_t^2} \left[ \frac{1}{N_t(i)} + \theta \frac{\partial p_t(i)}{\partial N_t(i)} - \theta \frac{\partial P_t}{\partial N_t(i)} + \Phi_t^f \frac{1}{N_t(i)} \right]
\]
(A.15)
The next step is to find \( \frac{\partial p_t(i)}{\partial N_t(i)} \). Combining (A.10) and (A.11):
\[
\frac{\partial p_t(i)}{\partial N_t(i)} = \mu_t(i) \frac{\mu_t(i) - 1}{1 - \epsilon_t(i)} mc_t(i) \left( (1 - \theta) \frac{\epsilon_t(i)}{p_t(i)} \frac{\partial p_t(i)}{\partial N_t(i)} + (\theta - 1) \frac{\epsilon_t(i)}{P_t} \frac{\partial P_t}{\partial N_t(i)} \right) + \mu_t(i) \frac{\partial mc_t(i)}{\partial N_t(i)}
\]
Applying symmetry with \( \epsilon_t(i) = 1/M_t \) and some algebra gives
\[
\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{M_t + (\mu_t - 1)(\theta - 1)}{1 + (\mu_t - 1)(\theta - 1)} \frac{(1 - \eta) \Phi_t^f}{\mu_t^2} \frac{\partial P_t}{\partial N_t(i)} < 0.
\]
(A.17)
Hence, \( \frac{\partial p_t(i)}{\partial N_t(i)} < \frac{\partial P_t}{\partial N_t(i)} < 0 \). From (A.10) and (A.11) it is now clear that \( \frac{\partial \epsilon_t(i)}{\partial N_t(i)} > 0 \) and \( \frac{\partial mc_t(i)}{\partial N_t(i)} < 0 \). An expansion of the product scope reduces the prices of the firm’s varieties and increases its market share. This stands in contrast to Pavlov and Weder (2017) where due to the love of variety, the firm would increase its prices. Using (A.10), under symmetry (A.9) can be written as
\[
\frac{\partial \pi_t(i)}{\partial N_t(i)} = \left( 1 - \eta + \eta \theta \left( 1 - \frac{1}{\mu_t} \right)^2 \right) \left( (1 - \theta) \frac{\partial p_t(i)}{\partial N_t(i)} + (\theta - 1) \frac{\partial P_t}{\partial N_t(i)} \right)
\]
\[
+ \left( 1 - \frac{\eta}{\mu_t} \right) \frac{\partial P_t}{\partial N_t(i)} - (\mu_t - \eta) \frac{\partial mc_t(i)}{\partial N_t(i)} - \eta \phi \frac{1 - M_t}{\mu_t Y_t} = 0.
\]
Finally, (A.15), (A.16), (A.17), and (10) are used in the above to solve for the product scope:
\[
N_t = \frac{1 - \eta \mu_t Y_t}{\eta \phi M_t} \left[ \frac{1 + \Phi_t^f}{\mu_t} \left( 1 - \frac{1}{M_t} \right) - \frac{M_t - 1}{M_t(\theta(M_t - 1) - \theta)} \right]
\]
(A.18)
Reminiscent of Pavlov and Weder (2017), the big term in square brackets is less than one and is increasing in \( M_t \) (converging to unity as the number of firms becomes very large).
An increase in the firm’s product scope reduces its marginal costs and prices. Other firms respond by reducing their prices and to lower this price competition firms under-expand their product scopes relative to the case of monopolistic competition where such strategic interactions are absent. This strategic effect diminishes as the number of firms increases and this gives an incentive to introduce new varieties. Recall the share of fixed costs in final output:

$$\Phi_t = \frac{\phi M_t N_t + \phi_f M_t}{Y_t} = \frac{\mu_t}{\eta} - 1.$$ 

Then, $$\Phi_t = \Phi^F_t + \Phi^v_t$$ together with (A.18) solves for $$\Phi^F_t \equiv \phi_f M_t / Y_t$$ and $$\Phi^v_t \equiv \phi M_t N_t / Y_t$$. As the number of firms becomes very large these cost shares approach the levels in the monopolistic competition version of the model (see Appendix A.3). As the markup is countercyclical, it is clear that $$\partial \Phi / \partial M < 0$$. It can also be shown that $$\partial \Phi^F / \partial M < 0$$ and $$\partial \Phi^v / \partial M > 0$$. Firm entry leads to an expansion of product scopes and increases the variety level fixed costs as a fraction of total output.

### A.3 Monopolistic competition

This Appendix shows that under monopolistic competition, markups and the product scope are constant over the business cycle. When firms are too small to influence the aggregate price index, $$P_t$$, the last term in (A.1) is absent and the markup is constant at $$\mu = \theta / (\theta - 1)$$. Profits can be written as

$$\pi_t(i) = \frac{\mu - \eta}{\mu} P_t Y_t \epsilon_t(i) - mc_t(i) \eta (N_t(i) \phi + \phi_f)$$

where the market share is

$$\epsilon_t(i) = \left( \frac{mc_t(i) \mu}{P_t} \right)^{1-\theta} M_t^{-1}$$

The first-order condition is

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = - \left[ \frac{\theta \mu - \eta}{\mu} P_t Y_t \epsilon_t(i) + \eta (N_t(i) \phi + \phi_f) \right] \frac{\partial mc_t(i)}{\partial N_t(i)} - mc_t(i) \eta \phi = 0$$

and from (A.5)

$$\frac{\partial mc_t(i)}{\partial N_t(i)} = - \left( \frac{y_t(i)}{N_t(i) + \phi_f N_t(i)^2} \right) mc_t(i)$$
Clearly, if $0 < \eta < 1$, then $\frac{\partial p_t(i)}{\partial N_t(i)} + \frac{\partial m_t(i)}{\partial N_t(i)} < 0$. As in the previous section, $\frac{\phi M_t N_t + \phi_f M_t}{Y_t} = \frac{\mu}{\eta} - 1$ is obtained from the zero profit condition. Then, putting these together under symmetry and some algebra gives

$$\Phi^p_t = \frac{\theta \left( \frac{\mu}{\eta} - 1 \right) \left( 1 + \Phi^f_t \right)}{\frac{\mu}{1-\eta} + \theta}$$

(A.19)

where once again $\Phi^f_t \equiv \frac{\phi_f M_t}{Y_t}$ and $\Phi^p_t \equiv \frac{\phi M_t N_t}{Y_t}$. With constant markups and zero profits each period, these cost shares are constant each period. Using (A.19) and $\Phi^f_t + \Phi^p_t = \frac{\mu}{\eta} - 1$ then gives $\Phi^f_t = \mu - 1$ and $\Phi^p_t = \frac{\mu}{\eta} - \mu$. Since $\Phi^f_t$ is constant, the number of firms is proportional to final output and the product scope is constant:

$$N = \frac{1 - \eta \mu Y_t}{\eta} \frac{Y_t}{\phi M_t} = \frac{1 - \eta}{\eta} \frac{\mu}{\mu - 1} \frac{\phi_f}{\phi}.$$

Parameter $\eta$ has no effect on local dynamics as output per firm and output per variety are constant. The dynamics of the model are identical to the constant markup mono-product model in Pavlov and Weder (2012) without the love of variety effects. Hence, indeterminacy cannot arise in this version of the model.

### A.4 Data sources

This Appendix details the source and construction of the U.S. data used in Section 5. All data is quarterly and for the period 1967:I-2010:IV.

1. Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

2. Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Durable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Gross Private Domestic Investment. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.


8. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

9. GDP Deflator = (5)/(6).

10. Real Per Capita Consumption, \( C_t = [(1) + (2)]/(9)/(8) \).

11. Real Per Capita Investment, \( X_t = [(3) + (4)]/(9)/(8) \).

12. Real Per Capita Output, \( Y_t = (10) + (11) \).

13. Per Capita Hours Worked, \( H_t = (7)/(8) \).


15. Total Private Closings, Number of Establishments, Rate (Percent). Source: Bureau of Labor Statistics, Series Id: BDS0000000000000000120006LQ5.

16. Net business formation, \((14) - (15)\).